AD-A274 548

(2)

ARMY RESEARCH LABORATORY

Computer Program for Numerical Evaluation of the Performance of an Overmoded TM_{01} Circular to TE_{10} Rectangular Waveguide Mode Converter

by Joseph R. Mautz and Roger F. Harrington

ARL-CR-129

December 1993

Approved for public release; distribution unlimited.

Editor's note: Although this report has been checked for completeness, it was not reviewed by the ARL Technical Publishing Branch for editorial correctness and conformance to the current standard, *Scientific and Technical Reports—Organization, Preparation, and Production*, American National Standards for Information Sciences (ANSI Z39.18-1987).

REPORT DOCUMENTATION PAGE

Form Approved OMB No. 0704-0188

1. AGENCY USE ONLY *(Leave blank)*

2. REPORT DATE
December 1993

3. REPORT TYPE AND DATES COVERED
Final, from August 91 to August 93

4. TITLE AND SUBTITLE
Computer Program for Numerical Evaluation of the Performance of an Overmoded TM_{01} Circular to TE_{10} Rectangular Waveguide Mode Converter

5. FUNDING NUMBERS
DA PR: 319863
PE: 63737
Contract: DAAL02-90-K-0027

6. AUTHOR(S)
Joseph R. Mautz and Roger F. Harrington

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
Department of Electrical and Computer Engineering
Syracuse University
Syracuse, NY 13244-1240

8. PERFORMING ORGANIZATION REPORT NUMBER
TR-93-1

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
U.S. Army Research Laboratory
Attn: AMSRL-WT-NH
2800 Powder Mill Road
Adelphi, MD 20783-1197

10. SPONSORING/MONITORING AGENCY REPORT NUMBER
ARL-CR-129

11. SUPPLEMENTARY NOTES
AMS code: 623737.00000
ARL proj: FE4027, 1FE427

12a. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release; distribution unlimited.

12b. DISTRIBUTION CODE

13. ABSTRACT *(Maximum 200 words)*

Two identical rectangular waveguides are fed by means of two symmetrically placed apertures in a circular waveguide. Several modes can propagate in the circular waveguide but only the TE_{10} mode can propagate in the rectangular waveguides. The rectangular waveguides have identical dimensions but may be terminated with different loads. Power is transmitted from the propagating modes in the circular waveguide to the TE_{10} mode in each rectangular waveguide. A computer program was written to evaluate this power and the magnitude of each orthogonal component of the tangential electric field in the apertures. This program is obtained by modifying an existing program. The modifications are described and listed. Sample input and output data are provided.

14. SUBJECT TERMS
Mode converter, method of moments, TM_{01} mode, TE_{10} mode

15. NUMBER OF PAGES
139

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
Unclassified	Unclassified	Unclassified	UL

NSN 7540-01-280-5500

Standard Form 298 (Rev. 2-89)
Prescribed by ANSI Std. Z39-18
298-102

Contents

Chapter 1

Introduction

Chapter 1 contains a statement of the waveguide mode converter problem. This problem is that treated in [1–3] generalized to the case where an arbitrary number or waveguide modes can propagate in the circular waveguide and where the excitation is an arbitrary mixture of the forward traveling waves of these modes.† The general method of solution is the same as that described in Section 1.2 of [3]. A computer program was written to implement this solution. This program was obtained by modifying the program that is described and listed in [3]. Chapter 2, which is based on Chapter 2 of [3], contains instructions for using the computer program. These instructions should enable one to use the program. In other words, they should enable one to modify the input data to suit his needs, to run the program with the modified input data. and to interpret the results.

The computer program consists of a main program, the subroutines MODES, BESIN, BES, INTERPOL, PHI, and DGN, the function subprogram FXY, and the subroutines DECOMP and SOLVE in that order. The subprograms MODES, BESIN, BES, INTERPOL, PHI, DGN, FXY, DECOMP, and SOLVE are exactly the same as those described and listed in Chapters 4 to 11 of [3]. The main program of the present report was obtained by modifying the main program in [3]. The formulas used in the main program to compute the excitation vector, the normalized coefficients of the propagating modes in each rectangular waveguide, the normalized coefficients of the propagating modes in the circular waveguide, and the two normalized

†In [1–3], only the even TM_{01} and even and odd TE_{11} modes can propagate in the circular waveguide, and the excitation is the forward traveling wave of the even TM_{01} mode. The amplitude of this wave is unity

orthogonal components of the electric field in each aperture are presented in Chapters 3 to 6, respectively. The differences between the main program of the present report and the main program in [3] are described in Chapter 7. The entire main program is listed in Chapter 7.

He who wants merely to use the computer program is advised to read only Chapters 1 and 2 of the present report and to consult Chapters 1 and 2 of [3] only when referred there. The user will have to go into Chapters 3 to 7 of the present report, and/or Chapters 3 to 11 of [3] only if he wants to modify the computer program or if unforeseen difficulty arises while running the program. If such difficulty does arise, the basic theory in [1] and [2] could be helpful.

1.1 Statement of the Problem

There is, as shown in Fig. 1, a circular waveguide that is closed at one end. Two symmetrically placed apertures in the lateral wall of this waveguide are backed by rectangular waveguides of identical dimensions but terminated with loads that may be different. The interiors of the left-hand rectangular waveguide, the right-hand rectangular waveguide, and the circular waveguide are called regions 1, 2, and 3, respectively. Homogeneous space of permeability μ and permittivity ϵ exists in all of these regions. The circular waveguide is of radius a and is terminated by a perfectly conducting wall at $z = L_3$. An arbitrary number of modes can propagate in the circular waveguide. Both rectangular waveguides run parallel to the x-axis. Both have the same cross section ($-\frac{b}{2} \leq y \leq \frac{b}{2}$, $-\frac{c}{2} \leq z \leq \frac{c}{2}$) where $c < b$ and b is such that only the TE_{10} dominant mode can propagate in each rectangular waveguide. As seen from the circular waveguide, the aperture A_1 which feeds the left-hand rectangular waveguide in Fig. 1 is the surface for which ($\rho = a$, $\pi - \phi_o \leq \phi \leq \pi + \phi_o$, $-\frac{c}{2} \leq z \leq \frac{c}{2}$) where ρ and ϕ are the cylindrical coordinates related to x and y by

$$\rho = \sqrt{x^2 + y^2} \tag{1.1}$$

$$\tan \phi = \frac{y}{x} \tag{1.2}$$

and

$$\phi_o = \sin^{-1}\left(\frac{b}{2a}\right). \tag{1.3}$$

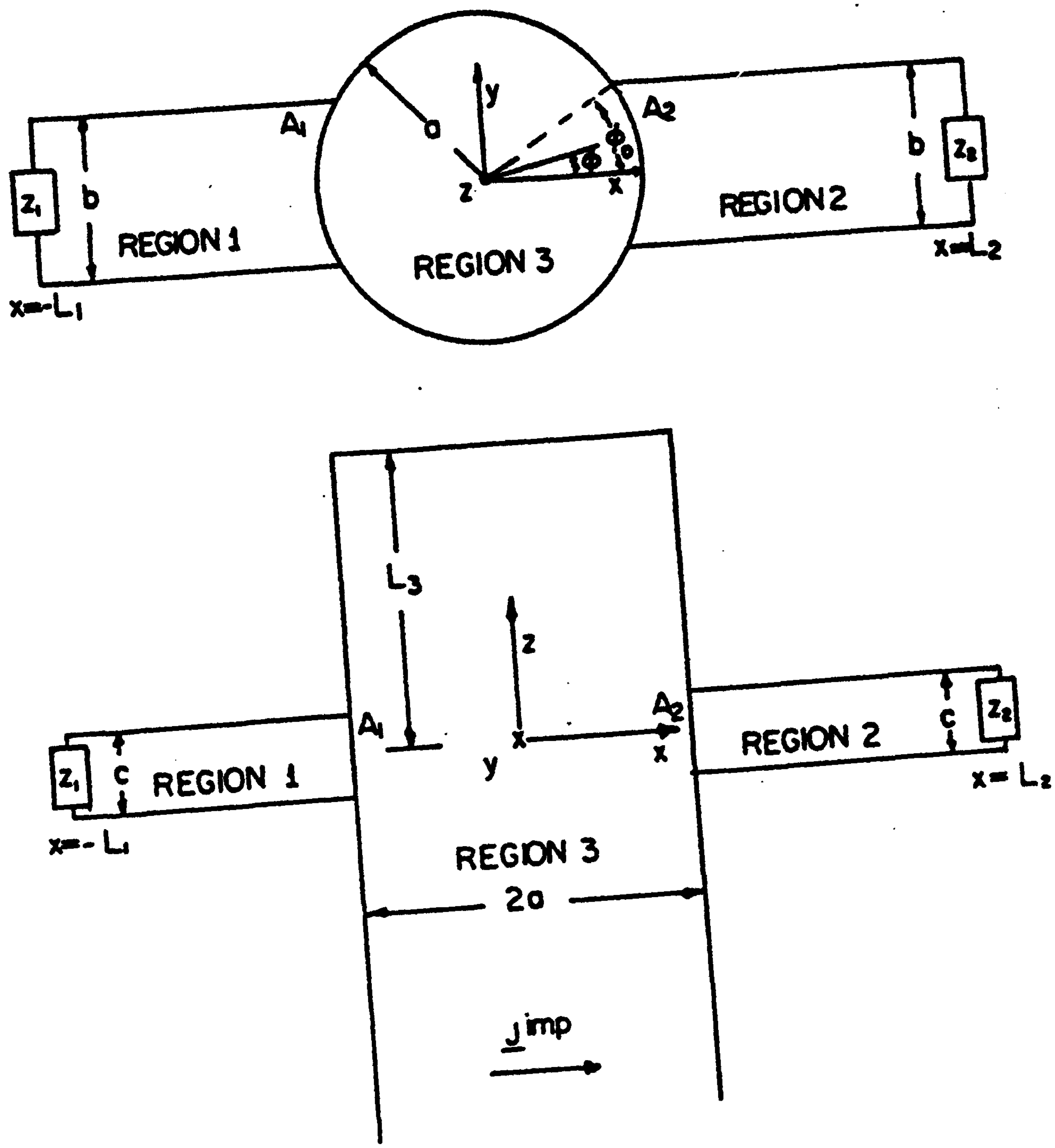

Fig. 1. Top and side views of the TM_{01} to TE_{10} mode converter.

As seen from the circular waveguide, the aperture A_2 which feeds the right-hand rectangular waveguide is the surface for which ($\rho = a$, $-\phi_o \leq \phi \leq \phi_o$, $-\frac{c}{2} \leq z \leq \frac{c}{2}$). However, as seen by the rectangular waveguides, the curved apertures A_1 and A_2 are approximated by plane apertures.

The excitation is most generally a mixture of the z-traveling (traveling in the z- direction) waves of all the propagating modes of the circular waveguide. This mixture is specified by giving the time-average power of each propagating mode and its phase at $z = L_3$. By assumption, the z-traveling waves at $z = -c/2$ are exactly this mixture. Concerning the fields in the part of region 3 for which $z > -c/2$ and in all of regions 1 and 2, it does not matter how this mixture of z-traveling waves is produced. We let it be produced by a sheet of electric current $\underline{J}^{imp}$ located at $z < -c/2$ in the circular waveguide. For simplicity, we assume that the circular waveguide is terminated at $z << -c/2$ by a matched load, i.e., any $-z$-traveling wave in the region for which $z < -c/2$ is never reflected.

The voltage to current ratio of the TE_{10} mode at $x = -L_1$ in region 1 is taken to be Z_1. All other rectangular waveguide modes are evanescent. The voltage to current ratios of the evanescent modes at $x = -L_1$ do not come into play because L_1 is taken to be so large that any evanescent wave emanating from the termination at $x = -L_1$ will have negligibly small amplitude upon arrival at aperture A_1. The voltage to current ratio of the TE_{10} mode at $x = L_2$ in region 2 is taken to be Z_2. Here, L_2 is taken to be so large that any evanescent wave emanating from the termination at $x = L_2$ will have negligibly small amplitude upon arrival at aperture A_2.

The problem is to find out how much time-average power of each propagating mode is reflected in the circular waveguide, how much time-average power is transmitted into the rectangular waveguides, and what are the magnitudes of the ϕ- and z-components of the electric field in the apertures A_1 and A_2.

1.2 Method of Solution

The method of solution is that described in Section 1.2 of [3]. In this method, the apertures are closed with infinitely thin perfect conductors and, as shown in Fig. 2, a magnetic current is placed on each side of each of these conductors.

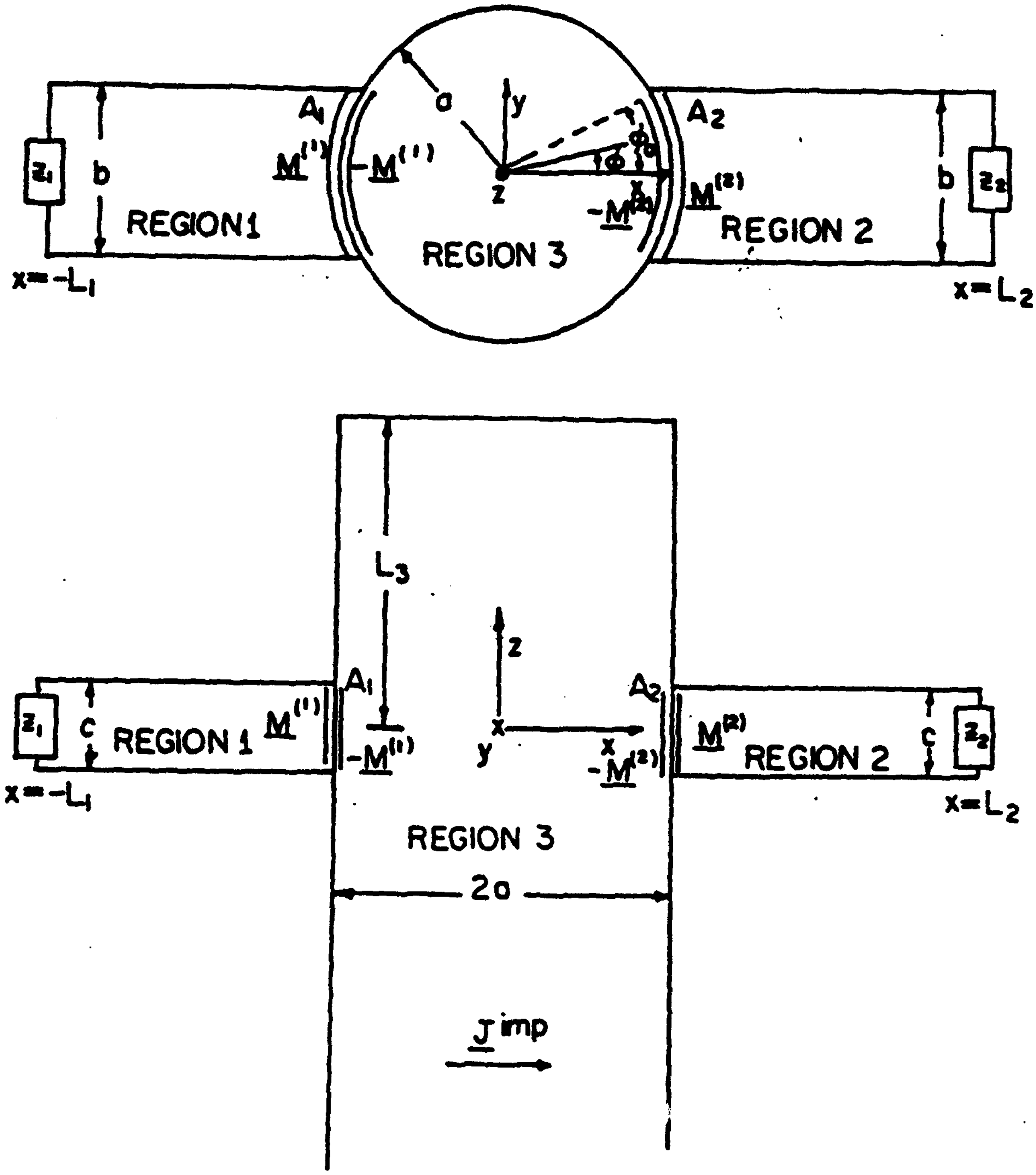

Fig. 2. Top and side views of the situation equivalent to that of Fig. 1.

Chapter 2

Instructions for Using the Computer Program

Written in FORTRAN, the computer program is available on diskette. On diskette, the computer program, which consists of a main program and some subprograms, is stored in the file DEC.92. Exemplary input data are stored in the file DEC92.DAT. These files are named DEC.92 and DEC92.DAT because December 1992 is the date of the present report, the report in which the computer program is described and listed. The main program, the subroutines MODES, BESIN, BES, INTERPOL, PHI, and DGN, the function subprogram FXY, and the subroutines DECOMP and SOLVE are stored in order in the file DEC.92

There are two modules of input data in the file DEC92.DAT. The first module of input data is preceded by the two comment statements

```
C       THIS IS THE FIRST MODULE OF INPUT DATA.
C       PUT IT IN THE FILE IN.DAT.
```

and followed by the comment statement

```
C       THIS IS THE SECOND MODULE OF INPUT DATA.
```

The second module of input data is preceded by the two comment statements

```
C       THIS IS THE SECOND MODULE OF INPUT DATA.
C       PUT IT IN THE FILE BESIN.DAT.
```

The last line of the second module of input data is the last line in the file DEC92.DAT.

To use the computer program, first follow the instructions given in the comment statements in the preceding paragraph, i.e., create input data files named IN.DAT and BESIN.DAT, put the first module of input data in IN.DAT, and put the second module of input data in BESIN.DAT. The data in IN.DAT are read by statements in the main program. The data in BESIN.DAT are read by statements in the subroutine BESIN. Next, create output data files named OUT.DAT and BESOUT.DAT. Then, give the command or commands necessary to run the computer program that resides in DEC.92. Running the program causes output data to be written in the output data files OUT.DAT and BESOUT.DAT. The data in OUT.DAT are written by statements in the main program. The data in BESOUT.DAT are written by statements in the subroutine BESIN.

The contents of the first module of input data that the user was instructed to take from the file DEC92.DAT and put in the file IN.DAT are merely exemplary data. One can modify the input data in the file IN.DAT to suit his needs.† The input data in the file IN.DAT are described in Section 2.1.1. The contents of the second module of input data that the user was instructed to take from the file DEC92.DAT and put in the file BESIN.DAT are, as stated in Section 2.1.2, not subject to change. If the program runs without difficulty, only the final output data need to be interpreted; intermediate output data can be ignored. The final output data are described in Section 2.2.1.

The first module of input data in the file DEC92.DAT is listed in Section 2.1.3. The second module of input data in the file DEC92.DAT is not listed in the present report: this module is exactly the same as the data listed under the heading "Listing of the second module of the sample input data" in Section 2.1.3 of [3]. The complete main program is listed in Chapter 7. However, the subprograms MODES, BESIN, BES, INTERPOL, PHI, DGN, FXY, DECOMP, and SOLVE are not listed in the present report because, except for replacement of PH1(100),PH2(100),PH3(100),PH4(100) by PH1(150),PH2(150),PH3(150),PH4(150) in the first common statement in the subroutine PHI, they are the same as in [3]. The output data that

†Modification of the input data in IN.DAT may require an increase in the storage area allocated to some arrays. Minimum allocations of arrays are given in Section 2.3.

are written in the file OUT.DAT when the computer program is run with all the input data exactly as taken from the file DEC92.DAT are listed in Section 2.2.2. The output data that are written in the file BESOUT.DAT are not listed in the present report because they are exactly the same as the output data described under the heading "Sample output written in the file BESOUT by statements in the subroutine BESIN" in Section 2.2.2 of [3].[†]

2.1 The Input Data

There are two modules of input data: the input data in the file IN.DAT and the input data in the file BESIN.DAT.

2.1.1 The Input Data in the File IN.DAT

The input data in the file IN.DAT are read from there by means of two groups of statements.

The first group of statements

The first group of statements that read input data from the file IN.DAT is

```
      READ(20,10) B,C,L1,L2,L3,BKM,XM,ZL1,ZL2
   10 FORMAT(4D14.7)
      READ(20,144) KAM,BKA0,DBKA,KE3M,NPHI,NZ
  144 FORMAT(I4,2D14.7,3I4)
      READ(20,146)(KE3(I),I=1,KE3M)
  146 FORMAT(15I4)
```

The above statements occur early in the main program. The data read in by them are the same as the data in Section 2.1.1 of [3]. The following description of these data is based on the description in Section 2.1.1 of [3].

[†]There is no file named BESOUT in [3]. The file BESOUT.DAT is meant. There are three instances in Section 2.2.2 of [3] where BESOUT is mistakenly written instead of BESOUT.DAT.

In the first of the above three read statements,

$$\mathrm{B} = \frac{b}{a} \tag{2.1}$$

$$\mathrm{C} = \frac{c}{a} \tag{2.2}$$

$$\mathrm{L1} = \frac{L_1}{a} \tag{2.3}$$

$$\mathrm{L2} = \frac{L_2}{a} \tag{2.4}$$

$$\mathrm{L3} = \frac{L_3}{a}. \tag{2.5}$$

The fields in the rectangular waveguides are expanded as linear combinations of all rectangular waveguide modes whose cutoff wavenumbers do not exceed BKM/b. The cutoff wavenumber of both the TM_{pq} and TE_{pq} rectangular waveguide modes is k_{pq} given by (see eq. (A.8) of [1])

$$k_{pq} = \sqrt{\left(\frac{p\pi}{b}\right)^2 + \left(\frac{q\pi}{c}\right)^2}. \tag{2.6}$$

Hence, the values of p and q are limited by

$$\sqrt{(p\pi)^2 + \left(\frac{q\pi b}{c}\right)^2} \leq \mathrm{BKM}. \tag{2.7}$$

The field in the circular waveguide is expanded as a linear combination of a finite number of circular waveguide modes. This number is controlled by the input variable XM. The even and odd TM_{rs} and the even and odd TE_{rs} circular waveguide modes are taken for all nonnegative integer values of r and s such that at least one of the values x_{rs} and x'_{rs} does not exceed XM. Actually, the odd TM_{rs} mode and the odd TE_{rs} mode exist only for $r \geq 1$. Here, x_{rs} is the s^{th} root of the Bessel function J_r, and x'_{rs} is the s^{th} root of J'_r where J'_r is the derivative of J_r with respect to its argument. If both of the values x_{01} and x'_{01} exceed XM, then execution is aborted because XM is deemed to be too small. The cutoff wavenumber k^{TM}_{rs} of the even and

odd TM_{rs} circular waveguide modes and the cutoff wavenumber k_{rs}^{TE} of the even and odd TE_{rs} circular waveguide modes are given by

$$k_{rs}^{\mathrm{TM}} = \frac{x_{rs}}{a} \tag{2.8}$$

$$k_{rs}^{\mathrm{TE}} = \frac{x'_{rs}}{a}. \tag{2.9}$$

Whereas the previously described first seven variables in the first read statement are real, the last two variables in this read statement are complex. These complex variables are ZL1 and ZL2 given by

$$\mathrm{ZL1} = Z_1 Y_{10}^{\mathrm{TE}} \tag{2.10}$$

$$\mathrm{ZL2} = Z_2 Y_{10}^{\mathrm{TE}} \tag{2.11}$$

where Z_1 and Z_2 are the impedance loads at $x = -L_1$ and $x = L_2$ in Fig. 2 and Y_{10}^{TE} is the characteristic admittance of the TE_{10} rectangular waveguide mode. Equations (2.28) and (2.29) of [3] are

$$\mathrm{ZL1} = \frac{-jZ_1}{\eta}\sqrt{\left(\frac{\pi}{kb}\right)^2 - 1} \tag{2.12}$$

$$\mathrm{ZL2} = \frac{-jZ_2}{\eta}\sqrt{\left(\frac{\pi}{kb}\right)^2 - 1} \tag{2.13}$$

where η is the intrinsic impedance given by

$$\eta = \sqrt{\frac{\mu}{\epsilon}}. \tag{2.14}$$

The first seven variables in the first read statement are obviously dimensionless. The variables ZL1 of (2.12) and ZL2 of (2.13) are dimensionless because Z_1/η and Z_2/η are dimensionless. Therefore, all the variables in the first read statement are dimensionless.

The variables KAM, BKA0, and DBKA in the second read statement are such that the waveguide mode converter problem is solved KAM times, once for each of the following values of ka:

$$ka = \mathrm{BKA0} + (\mathrm{KA} - 1) * \mathrm{DBKA}, \quad \mathrm{KA} = 1, 2, 3, \cdots, \mathrm{KAM}. \tag{2.15}$$

Here, k is the propagation constant and a is the radius of the circular waveguide.

The variables KE3M, NPHI, and NZ in the second read statement and $\{\text{KE3(I)}, \text{I} = 1, 2, \cdots, \text{KE3M}\}$ in the third read statement control the calculation and the writing out of the magnitudes of the ϕ- and z-components of the normalized electric field in the apertures A_1 and A_2. These magnitudes are

$$\frac{|E_\phi^{(A1)}|}{|\underline{E}^{(3+)}(\underline{J}^{imp}, \underline{0})|_{rms}}, \quad \frac{|E_z^{(A1)}|}{|\underline{E}^{(3+)}(\underline{J}^{imp}, \underline{0})|_{rms}}, \tag{2.16}$$

$$\frac{|E_\phi^{(A2)}|}{|\underline{E}^{(3+)}(\underline{J}^{imp}, \underline{0})|_{rms}}, \quad \frac{|E_z^{(A2)}|}{|\underline{E}^{(3+)}(\underline{J}^{imp}, \underline{0})|_{rms}} \tag{2.17}$$

where $|\underline{E}^{(3+)}(\underline{J}^{imp}, \underline{0})|_{rms}$ is the root mean square value of $|\underline{E}^{(3+)}(\underline{J}^{imp}, \underline{0})|$ over the cross section of the circular waveguide at $z = 0$. Here, $\underline{E}^{(3+)}(\underline{J}^{imp}, \underline{0})$ is the electric field of the z-traveling waves that would exist in the circular waveguide if the apertures were closed. In (2.16), $E_\phi^{(A1)}$ is the ϕ-component of the electric field at

$$(\phi, z) = \left(\left\{\pi + \phi_o\left(-1 + 2\frac{\text{J} - 1}{\text{NPHI} - 1}\right), \text{J} = 1, 2, \cdots, \text{NPHI}\right\}, 0\right) \tag{2.18}$$

in aperture A_1, and $E_z^{(A1)}$ is the z-component of the electric field at

$$(\phi, z) = \left(\pi, \left\{\frac{c}{2}\left(-1 + 2\frac{\text{J} - 1}{\text{NZ} - 1}\right), \text{J} = 1, 2, \cdots, \text{NZ}\right\}\right) \tag{2.19}$$

in aperture A_1. In (2.17), $E_\phi^{(A2)}$ is the ϕ-component of the electric field at

$$(\phi, z) = \left(\left\{\phi_o\left(-1 + 2\frac{\text{J} - 1}{\text{NPHI} - 1}\right), \text{J} = 1, 2, \cdots, \text{NPHI}\right\}, 0\right) \tag{2.20}$$

in aperture A_2, and $E_z^{(A2)}$ is the z-component of the electric field at

$$(\phi, z) = \left(0, \left\{\frac{c}{2}\left(-1 + 2\frac{\text{J} - 1}{\text{NZ} - 1}\right), \text{J} = 1, 2, \cdots, \text{NZ}\right\}\right) \tag{2.21}$$

in aperture A_2.

The quantities $\{KE3(I), I = 1, 2, \cdots, KE3M\}$ are positive integers such that

$$KE3(1) < KE3(2) < KE3(3) \cdots < KE3(KE3M - 1) < KE3(KE3M) \qquad (2.22)$$

where KE3(KE3M) is equal to either KAM or KAM + 1. The aperture field magnitudes (2.16) and (2.17) are calculated and printed out only for the values of ka of (2.15) for which

$$KA = \{KE3(1), KE3(2), KE3(3), \cdots, KE3(KAE)\} \qquad (2.23)$$

where

$$KAE = \begin{cases} KE3M - 1, & KE3(KE3M) = KAM + 1 \\ KE3M, & KE3(KE3M) = KAM. \end{cases} \qquad (2.24)$$

The second group of statements

The second group of statements that read input data from the file IN.DAT is

```
      DO 48 KA=1,KAM
      READ(20,109) RTMR,RTER
  109 FORMAT(10I4)
      READ(20,109)(STMR(R),R=1,RTMR)
      READ(20,109)(STER(R),R=1,RTER)
      DO 156 R=1,RTMR
      SMAX=STMR(R)
      IF(SMAX.EQ.0) GO TO 156
      R1=R-1
      READ(20,164)(BTME(S,R),S=1,SMAX)
  164 FORMAT(4E14.7)
      IF(R1.EQ.0) GO TO 156
      READ(20,164)(BTMO(S,R),S=1,SMAX)
  156 CONTINUE
      DO 167 R=1,RTER
      SMAX=STER(R)
      IF(SMAX.EQ.0) GO TO 167
```

```
      R1=R-1
      READ(20,164)(BTEE(S,R),S=1,SMAX)
      IF(R1.EQ.0) GO TO 167
      READ(20,164)(BTEO(S,R),S=1,SMAX)
  167 CONTINUE
   48 CONTINUE
```

The index KA of DO loop 48 in the above group of FORTRAN statements obtains ka of (2.15). The first statement inside DO loop 48 in the above group of statements occurs at about 540 lines into the main program.

The quantities BTME($s,r+1$), BTMO($s,r+1$), BTEE($s,r+1$), and BTEO($s,r+1$) indicate that the excitation of the circular waveguide includes the z-traveling waves whose electric fields are

$$\left(\frac{\mathrm{BTME}(s,r+1)e^{j\beta_{rs}^{TM}L_3}}{\sqrt{Z_{rs}^{TMeo}}}\right)E_{rs}^{TMe+}, \left(\frac{\mathrm{BTMO}(s,r+1)e^{j\beta_{rs}^{TM}L_3}}{\sqrt{Z_{rs}^{TMeo}}}\right)E_{rs}^{TMo+}, \quad (2.25)$$

$$\left(\frac{\mathrm{BTEE}(s,r+1)e^{j\beta_{rs}^{TE}L_3}}{\sqrt{Y_{rs}^{TEeo}}}\right)E_{rs}^{TEe+}, \left(\frac{\mathrm{BTEO}(s,r+1)e^{j\beta_{rs}^{TE}L_3}}{\sqrt{Y_{rs}^{TEeo}}}\right)E_{rs}^{TEo+} \quad (2.26)$$

where E_{rs}^{TMe+}, E_{rs}^{TMo+}, E_{rs}^{TEe+}, and E_{rs}^{TEo+} are the electric fields of the z-traveling even TM_{rs} wave, the z-traveling odd TM_{rs} wave, the z-traveling even TE_{rs} wave, and the z-traveling odd TE_{rs} wave, respectively. These electric fields are given by eqs. (B.1), (B.26), (B.35), and (B.55) of [1]. Furthermore, β_{rs}^{TM} and Z_{rs}^{TMeo} are, respectively, the wavenumber and the characteristic impedance of the TM_{rs} mode. Moreover, β_{rs}^{TE} and Y_{rs}^{TEeo} are, respectively, the wavenumber and the characteristic admittance of the TE_{rs} mode.

The range of values of r and s in (2.25) and (2.26) should cover all the propagating modes in the circular waveguide. However, he who does not know which modes propagate in the circular waveguide will not know what the maximum values of r and s are in (2.25) and (2.26). Hence, he will not know what the values of RTMR, RTER, {STMR(R), R = 1, RTMR} and {STER(R), R = 1, RTER} should be. Concerning the quantities in the previous sentence, the following action is taken in the computer program. If one or more of these quantities is too small so that the the range of values of r and s in (2.25) and (2.26) does not cover all the propagating modes, then

the coefficients of the missing propagating modes are set equal to zero. If one or more of the quantities mentioned in the third from the last sentence is too large so that nonpropagating modes are included in (2.25) and (2.26), then these nonpropagating modes are ignored.

2.1.2 The Input Data in the File BESIN.DAT

The input data in the file BESIN.DAT are read from there by means of statements in the subroutine BESIN. One who merely uses the computer program does not have to concern oneself with these input data because he will never have to change any of them.

2.1.3 The Exemplary Input Data

The exemplary input data consist of the two modules of input data in the file DEC92.DAT. The user was instructed to put the first module in the file IN.DAT and to put the second module in the file BESIN.DAT.

The first module of input data is for the structure of Fig. 2 with

$$\frac{b}{a} = 1.1 \tag{2.27}$$

$$\frac{c}{a} = 0.5 \tag{2.28}$$

$$\frac{L_3}{a} = 1.838694 \tag{2.29}$$

$$Z_1 = \frac{1}{Y_{10}^{\mathrm{TE}}} \tag{2.30}$$

$$Z_2 = \frac{1}{Y_{10}^{\mathrm{TE}}} \tag{2.31}$$

$$ka = 5.6. \tag{2.32}$$

The value of L_3/a in (2.29) was, for want of anything better, taken from eq. (2.49) of [3]. In (2.30) and (2.31), Y_{10}^{TE} is the characteristic admittance of the TE_{10} rectangular waveguide mode so that Z_1 and Z_2 are matched loads.

Consequently, the electromagnetic field in the circular waveguide will not depend on either L_1 or L_2 provided that, as assumed in Chapter 1 of [1], L_1 and L_2 are large enough so that any evanescent wave emanating from either the termination at $z = -L_1$ or that at $z = L_2$ will have negligibly small amplitude upon arrival at the pertinent aperture in the circular waveguide.

The value of ka in (2.32) was obtained by setting

$$\text{KAM} = 1 \tag{2.33}$$

$$\text{BKA0} = 5.6 \tag{2.34}$$

$$\text{DBKA} = 0.0. \tag{2.35}$$

The values of the variables BKM, XM, NPHI, NZ, KE3M, and KE3(1) in the exemplary input data are

$$\text{BKM} = 15.0 \tag{2.36}$$

$$\text{XM} = 40.0 \tag{2.37}$$

$$\text{NPHI} = 81 \tag{2.38}$$

$$\text{NZ} = 21 \tag{2.39}$$

$$\text{KE3M} = 1 \tag{2.40}$$

$$\text{KE3}(1) = 1. \tag{2.41}$$

Because of the values of KE3M and KE3(1) given by (2.40) and (2.41), the magnitudes of the ϕ- and z-components of the normalized electric field in the apertures A_1 and A_2 evaluated at $ka = 5.6$ will appear in the output data.

For $ka = 5.6$,

$$\text{RTMR} = 2 \tag{2.42}$$

$$\text{RTER} = 3 \tag{2.43}$$

$$\left.\begin{aligned} \text{STMR}(1) &= 2 \\ \text{BTME}(1,1) &= 1.0 + j0.0 \\ \text{BTME}(2,1) &= 0.9 + j0.0 \end{aligned}\right\} \tag{2.44}$$

$$\left.\begin{aligned} \text{STMR}(2) &= 1 \\ \text{BTME}(1,2) &= 0.8 + j0.1 \\ \text{BTMO}(1,2) &= 0.7 + j0.15 \end{aligned}\right\} \tag{2.45}$$

$$\left.\begin{aligned} \text{STER}(1) &= 1 \\ \text{BTEE}(1,1) &= 0.6 + j0.2 \end{aligned}\right\} \tag{2.46}$$

$$\left.\begin{aligned} \text{STER}(2) &= 2 \\ \text{BTEE}(1,2) &= 0.1 + j0.8 \\ \text{BTEE}(2,2) &= 0.15 + j0.7 \\ \text{BTEO}(1,2) &= 0.2 + j0.6 \\ \text{BTEO}(2,2) &= 0.5 + j0.3 \end{aligned}\right\} \tag{2.47}$$

$$\left.\begin{aligned} \text{STER}(3) &= 1 \\ \text{BTEE}(1,3) &= 0.4 + j0.4 \\ \text{BTEO}(1,3) &= 0.3 + j0.5 \end{aligned}\right\}. \tag{2.48}$$

The values of the B's in (2.44)–(2.48) were arbitrarily chosen.

Only the first module of input data in the file DEC92.DAT is listed below. The second module of input data in the file DEC92.DAT is exactly the same as listed in Section 2.1.3 of [3].

Listing of the first module of input data in the file DEC92.DAT

```
0.1100000D+01 0.5000000D+00 0.4000000D+02 0.4000000D+02
0.1838694D+01 0.1500000D+02 0.4000000D+02 0.1000000D+01
0.0000000D+00 0.1000000D+01 0.0000000D+00
  1 0.5600000D+01 0.0000000D+00   1  81  21
  1
  2   3
  2   1
  1   2   1
0.1000000E+01 0.0000000E+00 0.9000000E+00 0.0000000E+00
0.8000000E+00 0.1000000E+00
0.7000000E+00 0.1500000E+00
0.6000000E+00 0.2000000E+00
0.1000000E+00 0.8000000E+00 0.1500000E+00 0.7000000E+00
0.2000000E+00 0.6000000E+00 0.5000000E+00 0.3000000E+00
0.4000000E+00 0.4000000E+00
0.3000000E+00 0.5000000E+00
```

2.2 The Output Data

The output data consist of the input data, some intermediate output data, and the final output data. The input data were described in Sections 2.1.1 and 2.1.2. The final output data are described in Section 2.2.1. The intermediate output data are included in the exemplary output data listed and discussed in Section 2.2.2.

2.2.1 Description of the Final Output Data

The final output data consist of the normalized coefficients of the TE_{10} waves in the rectangular waveguides, the normalized coefficients of all the waves of all the propagating modes in the circular waveguide, the magnitudes of the ϕ and z components of the normalized electric field in the apertures, the electrical length ka, the time-average incident power, the time-average transmitted power, and the time-average reflected power.

The electric field of the TE_{10} waves in the left-hand rectangular waveguide in Fig. 2 is $[\underline{E}^{(1)}]_{\text{prop}}$ written as

$$\left[\underline{E}^{(1)}\right]_{\text{prop}} = \frac{(\text{C1OUT})\underline{E}_{10}^{\text{TE}-} e^{j\beta_{10}x_o} + (\text{C1IN})\underline{E}_{10}^{\text{TE}+} e^{-j\beta_{10}x_o}}{\sqrt{Y_{10}^{\text{TE}}}} \tag{2.49}$$

Here, $\underline{E}^{(1)}$ is the total electric field in the waveguide and the subscript "prop" denotes the part that consists of all the waves of the forward and backward traveling TE_{10} modes that propagate in the waveguide. In (2.49), $\underline{E}_{10}^{\text{TE}-}$ is the electric field of the $-z$-traveling TE_{10} mode, and $\underline{E}_{10}^{\text{TE}+}$ is the electric field of the z-traveling TE_{10} mode. These fields are given by eqs. (A.15) and (A.14) of [1]. The quantities β_{10} and Y_{10}^{TE} in (2.49) are, respectively, the wavenumber and the characteristic admittance of the TE_{10} rectangular waveguide modes. Also,

$$x_o = \frac{a \sin \phi_o}{\phi_o}. \tag{2.50}$$

The quantities C1OUT and C1IN in (2.49) are the normalized coefficients of the TE_{10} waves in the left-hand rectangular waveguide. The electric field of the TE_{10} waves in the right-hand rectangular waveguide in Fig. 2 is $[\underline{E}^{(2)}]_{\text{prop}}$

written as

$$\left[E^{(2)}\right]_{\text{prop}} = \frac{(\text{C2OUT})E_{10}^{\text{TE}+}e^{j\beta_{10}z_0} + (\text{C2IN})E_{10}^{\text{TE}-}e^{-j\beta_{10}z_0}}{\sqrt{Y_{10}^{\text{TE}}}} \tag{2.51}$$

The quantities C2OUT and C2IN in (2.51) are the normalized coefficients of the TE_{10} waves in the right-hand rectangular waveguide.

The coefficients C1OUT, C1IN, C2OUT, and C2IN in (2.49) and (2.51) are variables in the main program. They are, for ka = BKA, written out by means of the statements

```
   WRITE(21,69) C1OUT,C1IN
69 FORMAT('C1OUT=',2E14.7,', C1IN=',2E14.7)
   WRITE(21,70) C2OUT,C2IN
70 FORMAT('C2OUT=',2E14.7,', C2IN=',2E14.7)
```

Computer program variables C1OUTS, C1INS, C2OUTS, C2INS, and PT are defined by

```
C1OUTS=C1OUT*CONJG(C1OUT)
C1INS=C1IN*CONJG(C1IN)
C2OUTS=C2OUT*CONJG(C2OUT)
C2INS=C2IN*CONJG(C2IN)
PT=C1OUTS-C1INS+C2OUTS-C2INS
```

and printed out according to

```
   WRITE(21,82) C1OUTS,C1INS,C2OUTS,C2INS,PT
82 FORMAT('C1OUTS=',E14.7,', C1INS=',E14.7,', C2OUTS=',
  1E14.7/'C2INS=',E14.7,', PT=',E14.7)
```

The previous three groups of FORTRAN statements are inside DO loop 48 whose index KA obtains ka of (2.15). The quantities C1OUTS, C1INS, C2OUTS, and C2INS are, respectively, the time-average powers of the outgoing TE_{10} wave in the left-hand rectangular waveguide, the incoming TE_{10} wave in the left-hand rectangular waveguide, the outgoing TE_{10} wave in the right-hand rectangular waveguide, and the incoming TE_{10} wave in the left-hand rectangular waveguide. Here, "outgoing" means going away from the pertinent aperture, and "incoming" means coming in toward the pertinent

aperture. The quantity PT is the time-average power P_t transmitted into the rectangular waveguides. Here, P_t is given by

$$P_t = P_t^{(1)} + P_t^{(2)} \tag{2.52}$$

where $P_t^{(1)}$ and $P_t^{(2)}$ will be given by (4.7) and (4.8), respectively.

The electric field of all the waves of all the propagating modes in the circular waveguide is $[\underline{E}^{(3)}]_{\text{prop}}$ written as

$$\begin{aligned}
\left[\underline{E}^{(3)}\right]_{\text{prop}} &= \\
&\sum_{r=0}\sum_{s=1}\frac{(\text{BTME}(s,r+1))\underline{E}_{rs}^{\text{TMe}+}e^{j\beta_{rs}^{\text{TM}}L_3} - (\text{CTME})\underline{E}_{rs}^{\text{TMe}-}e^{-j\beta_{rs}^{\text{TM}}L_3}}{\sqrt{Z_{rs}^{\text{TMeo}}}} \\
&+\sum_{r=1}\sum_{s=1}\frac{(\text{BTMO}(s,r+1))\underline{E}_{rs}^{\text{TMo}+}e^{j\beta_{rs}^{\text{TM}}L_3} - (\text{CTMO})\underline{E}_{rs}^{\text{TMo}-}e^{-j\beta_{rs}^{\text{TM}}L_3}}{\sqrt{Z_{rs}^{\text{TMeo}}}} \\
&+\sum_{r=0}\sum_{s=1}\frac{(\text{BTEE}(s,r+1))\underline{E}_{rs}^{\text{TEe}+}e^{j\beta_{rs}^{\text{TE}}L_3} + (\text{CTEE})\underline{E}_{rs}^{\text{TEe}-}e^{-j\beta_{rs}^{\text{TE}}L_3}}{\sqrt{Y_{rs}^{\text{TEeo}}}} \\
&+\sum_{r=1}\sum_{s=1}\frac{(\text{BTEO}(s,r+1))\underline{E}_{rs}^{\text{TEo}+}e^{j\beta_{rs}^{\text{TE}}L_3} + (\text{CTEO})\underline{E}_{rs}^{\text{TEo}-}e^{-j\beta_{rs}^{\text{TE}}L_3}}{\sqrt{Y_{rs}^{\text{TEeo}}}}
\end{aligned} \tag{2.53}$$

Here, $\underline{E}^{(3)}$ is the total electric field in the circular waveguide, and the subscript "prop" denotes the part that consists of all the waves of all the propagating modes in the circular waveguide. In (2.53), $\underline{E}_{rs}^{\text{TMe}+}$ is the electric field of the z-traveling even TM_{rs} circular waveguide mode, $\underline{E}_{rs}^{\text{TMe}-}$ is that of the $-z$-traveling even TM_{rs} circular waveguide mode, $\underline{E}_{rs}^{\text{TMo}+}$ is that of the z-traveling odd TM_{rs} circular waveguide mode, $\underline{E}_{rs}^{\text{TMo}-}$ is that of the $-z$-traveling odd TM_{rs} circular waveguide mode, $\underline{E}_{rs}^{\text{TEe}+}$ is that of the z-traveling even TE_{rs} circular waveguide mode, $\underline{E}_{rs}^{\text{TEe}-}$ is that of the $-z$-traveling even TE_{rs} circular waveguide mode, $\underline{E}_{rs}^{\text{TEo}+}$ is that of the z-traveling odd TE_{rs} circular waveguide mode, and $\underline{E}_{rs}^{\text{TEo}-}$ is that of the $-z$-traveling odd TE_{rs} circular waveguide mode. These fields are given by eqs. (B.1), (B.2), (B.26), (B.27), (B.35), (B.36), (B.55), and (B.56) of [1]. The quantities β_{rs}^{TM} and Z_{rs}^{TMeo} in (2.53) are, respectively, the wavenumber and the characteristic

impedance of the TM_{rs} circular waveguide modes. Furthermore, β_{rs}^{TE} and Y_{rs}^{TEeo} are, respectively, the wavenumber and the characteristic admittance of the TE_{rs} circular waveguide modes. The double summations with respect to r and s in (2.53) are over all the propagating modes of the circular waveguide. The quantities BTME(s,r+1), CTME, BTMO(s,r+1), CTMO, BTEE(s,r+1), CTEE, BTEO(s,r+1), and CTEO in (2.53) are the normalized coefficients of all the waves of all the propagating modes in the circular waveguide. Each of these coefficients depends on r and s. The B's were read in; CTME, CTMO, CTEE, and CTEO will be given by (7.149), (7.155), (7.189), and (7.195), respectively.

The normalized coefficients CTME, CTMO, CTEE, and CTEO are variables in the main program. They and their magnitudes squared

```
CTMES=CTME*CONJG(CTME)
CTMOS=CTMO*CONJG(CTMO)
CTEES=CTEE*CONJG(CTEE)
CTEOS=CTEO*CONJG(CTEO)
```

are written out by the means of the statements

```
    DO 172 R=1,RTM
    SMAX=STM(R)
    IF(SMAX.EQ.0) GO TO 172
    R1=R-1
    DO 173 S=1,SMAX
    WRITE(21,160) R,S
160 FORMAT('R=',I4,', S=',I4)
    WRITE(21,162) CTME,CTMES
162 FORMAT('CTME=',2E14.7,', CTMES=',E14.7)
    IF(R1.EQ.0) GO TO 173
    WRITE(21,158) CTMO,CTMOS
158 FORMAT('CTMO=',2E14.7,', CTMOS=',E14.7)
173 CONTINUE
172 CONTINUE
    DO 180 R=1,RTE
    SMAX=STE(R)
    IF(SMAX.EQ.0) GO TO 180
    R1=R-1
```

```
      DO 181 S=1,SMAX
      WRITE(21,160) R,S
      WRITE(21,165) CTEE,CTEES
  165 FORMAT('CTEE=',2E14.7,', CTEES=',E14.7)
      IF(R1.EQ.0) GO TO 181
      WRITE(21,90) CTEO,CTEOS
   90 FORMAT('CTEO=',2E14.7,', CTEOS=',E14.7)
  181 CONTINUE
  180 CONTINUE
```

The above group of FORTRAN statements is inside DO loop 48 whose index KA obtains ka of (2.15). The quantities CTMES, CTMOS, CTEES, and CTEOS are the time-average powers of the waves of the $-z$-traveling even and odd TM_{rs} and even and odd TE_{rs} propagating modes of the circular waveguide. For example, CTMES is the time-average power of the wave of the $-z$-traveling even TM_{rs} mode in the circular waveguide.

Computer program variables E3A1PS(J), E3A1ZS(J), E3A2PS(J), and E3A2ZS(J) are defined by

$$\text{E3A1PS(J)} = \frac{\left|E_\phi^{(A1)}\right|}{\left|\underline{E}^{(3+)}(\underline{J}^{\text{imp}},\underline{0})\right|_{\text{rms}}}, \quad \begin{cases} \phi = \pi + \left(-1 + 2\frac{\text{J}-1}{\text{NPHI}-1}\right)\phi_o \\ z = 0 \end{cases} \tag{2.54}$$

$$\text{E3A1ZS(J)} = \frac{\left|E_z^{(A1)}\right|}{\left|\underline{E}^{(3+)}(\underline{J}^{\text{imp}},\underline{0})\right|_{\text{rms}}}, \quad \begin{cases} \phi = \pi \\ z = \left(-1 + 2\frac{\text{J}-1}{\text{NZ}-1}\right)\frac{c}{2} \end{cases} \tag{2.55}$$

$$\text{E3A2PS(J)} = \frac{\left|E_\phi^{(A2)}\right|}{\left|\underline{E}^{(3+)}(\underline{J}^{\text{imp}},\underline{0})\right|_{\text{rms}}}, \quad \begin{cases} \phi = \left(-1 + 2\frac{\text{J}-1}{\text{NPHI}-1}\right)\phi_o \\ z = 0 \end{cases} \tag{2.56}$$

$$\text{E3A2ZS(J)} = \frac{\left|E_z^{(A2)}\right|}{\left|\underline{E}^{(3+)}(\underline{J}^{\text{imp}},\underline{0})\right|_{\text{rms}}}, \quad \begin{cases} \phi = 0 \\ z = \left(-1 + 2\frac{\text{J}-1}{\text{NZ}-1}\right)\frac{c}{2} \end{cases} \tag{2.57}$$

where $|\underline{E}^{(3+)}(\underline{J}^{\text{imp}},\underline{0})|_{\text{rms}}$ is the root mean square value of the magnitude of the transverse electric field of the z-traveling waves that would exist over the cross section of the circular waveguide at $z = 0$ if the apertures were closed.

In (2.54) and (2.55), $|E_\phi^{(A1)}|$ and $|E_z^{(A1)}|$ are, respectively, the magnitudes of the ϕ- and z-components of the electric field at the indicated values of ϕ and z in the left-hand aperture in Fig. 2. In (2.56) and (2.57), $|E_\phi^{(A2)}|$ and $|E_z^{(A2)}|$ are, respectively, the magnitudes of the ϕ- and z-components of the electric field at the indicated values of ϕ and z in the right-hand aperture in Fig. 2. The right-hand sides of (2.54)–(2.57) will be the magnitudes of the right-hand sides of (6.6), (6.8), (6.7), and (6.9), respectively.

The dimensioned variables E3A1PS, E3A1ZS, E3A2PS, and E3A2ZS in the main program are written out by the statements

```
      WRITE(21,140)(E3A1PS(J),J=1,NPHI)
  140 FORMAT('E3A1PS'/(5E14.7))
      WRITE(21,141)(E3A1ZS(J),J=1,NZ)
  141 FORMAT('E3A1ZS'/(5E14.7))
      WRITE(21,142)(E3A2PS(J),J=1,NPHI)
  142 FORMAT('E3A2PS'/(5E14.7))
      WRITE(21,143)(E3A2ZS(J),J=1,NZ)
  143 FORMAT('E3A2ZS'/(5E14.7))
```

The above group of statements is inside DO loop 48 whose index KA obtains ka of (2.15). However, this group of statements is executed only for those values of KA on the right-hand side of (2.23).

The computer program variables BKAPLT(KA), PINC(KA), PTRAN (KA), and PREFL(KA) are set equal to ka, the time-average power of the z-traveling waves that would exist in the circular waveguide if the apertures were shorted, the time-average power transmitted into the rectangular waveguides, and the time-average power reflected in the circular waveguide, respectively. Here, ka is given in terms of KA by (2.15), and all these time-average powers are evaluated at this value of ka. The computer program variables cited in the first sentence of this paragraph are written out by the statements

```
      WRITE(21,149)(BKAPLT(I),I=1,KAM)
  149 FORMAT('BKAPLT'/(5E14.7))
      WRITE(21,189)(PINC(I),I=1,KAM)
  189 FORMAT('PINC'/(5E14.7))
      WRITE(21,92)(PTRAN(I),I=1,KAM)
   92 FORMAT('PTRAN'/(5E14.7))
```

```
      WRITE(21,139)(PREFL(I),I=1,KAM)
  139 FORMAT('PREFL'/(5E14.7))
```

All three of the variables PINC(I), PTRAN(I), and PREFL(I) are calculated independently so that satisfaction of the equation of conservation of time-average power

$$\mathrm{PINC(I)} = \mathrm{PTRAN(I)} + \mathrm{PREFL(I)} \tag{2.58}$$

gives some assurance that the values of these three variables are accurate.

2.2.2 The Exemplary Output Data

The output data that resulted when the computer program was run with the exemplary input data of Section 2.1.3 are presented in here in Section 2.2.2.

Listing of the output data written in the file OUT.DAT

```
B,C,L1,L2,L3,BKN,XN,ZL1,ZL2
 0.1100000D+01 0.5000000D+00 0.4000000D+02 0.4000000D+02
 0.1838694D+01 0.1500000D+02 0.4000000D+02 0.1000000D+01
 0.0000000D+00 0.1000000D+01 0.0000000D+00
KAM=    1, BKAO= 0.5600000D+01, DBKA= 0.0000000D+00
KE3M=    1, NPHI=  81, NZ=  21
KE3
   1
NMAX=    3
MM
   5   5   2
KTM=    5, KTE=  11, K1=  16
BKB= 0.6160000E+01
YREC
 0.1388117E+01 0.0000000E+00 0.8773033E+00 0.0000000E+00
 0.6202015E+00 0.0000000E+00 0.4756268E+00 0.0000000E+00
 0.4824891E+00 0.0000000E+00 0.0000000E+00-0.8601751E+00
-0.2009854E+00 0.0000000E+00-0.1157968E+01 0.0000000E+00
-0.1778083E+01 0.0000000E+00-0.5088007E+00 0.0000000E+00
-0.7204005E+00 0.0000000E+00-0.1139857E+01 0.0000000E+00
-0.1612379E+01 0.0000000E+00-0.2102489E+01 0.0000000E+00
-0.2008858E+01 0.0000000E+00-0.2072586E+01 0.0000000E+00
 0.1388117E+01 0.0000000E+00 0.8773033E+00 0.0000000E+00
 0.6202015E+00 0.0000000E+00 0.4756268E+00 0.0000000E+00
```

```
 0.4824891E+00 0.0000000E+00 0.0000000E+00-0.8801751E+00
-0.2009854E+00 0.0000000E+00-0.1157968E+01 0.0000000E+00
-0.1778083E+01 0.0000000E+00-0.5088007E+00 0.0000000E+00
-0.7204005E+00 0.0000000E+00-0.1139857E+01 0.0000000E+00
-0.1612379E+01 0.0000000E+00-0.2102489E+01 0.0000000E+00
-0.2008858E+01 0.0000000E+00-0.2072586E+01 0.0000000E+00
RTMR=   2, RTER=   3
STMR
   2   1
STER
   1   2   1
BTME(   1,S)
 0.1000000D+01 0.0000000D+00 0.9000000D+00 0.0000000D+00
BTME(   2,S)
 0.8000000D+00 0.1000000D+00
BTMO(   2,S)
 0.7000000D+00 0.1500000D+00
BTEE(   1,S)
 0.6000000D+00 0.2000000D+00
BTEE(   2,S)
 0.1000000D+00 0.8000000D+00 0.1500000D+00 0.7000000D+00
BTEO(   2,S)
 0.2000000D+00 0.6000000D+00 0.5000000D+00 0.3000000D+00
BTEE(   3,S)
 0.4000000D+00 0.4000000D+00
BTEO(   3,S)
 0.3000000D+00 0.5000000D+00
RTM=   3, RTE=   5
STM
   2   1   1
STE
   1   2   1   1   1
TI
 0.3518446E+00-0.3492765E+00-0.8924618E-01-0.9492077E-02
 0.4838421E-01-0.4796127E-01-0.2600871E-01-0.5229265E-02
 0.4249075E-01-0.2229798E+00-0.1420960E+00 0.1220089E+01
 0.1530048E-01 0.1602860E+00-0.7006625E-01 0.4109569E+00
 0.1169016E-01 0.7381111E-01-0.3507916E+00 0.4863083E+00
-0.3334311E-01-0.4703431E-01-0.2449838E-01-0.5968513E-01
 0.4846611E-01-0.5552906E-01-0.2782277E-01-0.2710812E-01
-0.3636899E+00 0.3768167E-01 0.1703688E-01-0.2801486E-01
 0.7564259E-01 0.6343706E+00 0.7946628E-01-0.1782045E+00
-0.4001319E-02 0.1398010E+00 0.2298670E-01-0.5430467E-01
-0.1800996E+00-0.3359992E-01 0.5744629E+00 0.3378020E+00
```

```
 0.2252414E+00-0.4967495E-01 0.1391609E+00 0.1533509E+00
 0.1054543E+00-0.1995303E-01 0.1768541E+00-0.4615320E+00
-0.1098981E+00 0.3791547E+00 0.2804974E-01-0.1124333E+00
-0.1562852E-01 0.1957393E+00 0.2683454E-01-0.8176542E-01
-0.5237414E+00 0.1198875E+00 0.1410975E+00-0.1459439E+00
V
 0.1513356E+00-0.6882286E-01-0.3940935E-01-0.5829981E-02
 0.7838906E-01-0.6548309E-01-0.3753356E-01 0.2704030E-01
-0.1063785E+00-0.1806047E+00-0.8220977E+00-0.2998156E+00
-0.3207238E+00-0.6738196E-01 0.6085703E-01-0.1743149E+00
-0.3028706E-01-0.1032381E-01 0.2622715E+00-0.6327962E+00
-0.9517164E-01 0.1039968E+00 0.4132394E-01-0.3916366E-01
-0.2333756E-01 0.8843917E-02 0.1418404E-01-0.1133910E-01
 0.1169696E+00-0.2599828E-01-0.4410913E-01 0.3505990E-01
 0.5192810E-01 0.2841841E+00 0.2304141E-01-0.3864313E-01
 0.5688362E-02 0.1528783E+00 0.1899074E-01-0.7124534E-01
-0.2613861E+00 0.2213530E-02-0.1497400E+00 0.3608209E+00
-0.4099245E+00 0.2612040E+00-0.5671563E-01-0.6756795E-01
-0.4167582E-01 0.2390368E-01-0.5105329E-01 0.5105926E+00
 0.6057863E-01-0.3309209E+00-0.2599351E-01 0.1540932E+00
 0.7602720E-02-0.8716582E-01-0.7560617E-02 0.4033007E-01
 0.1545046E+00-0.6906327E-01-0.5297850E-01 0.4250777E-01
C1OUT=-0.7624597E+00-0.2780659E+00, C1IN= 0.0000000E+00 0.0000000E+00
C2OUT=-0.1388773E+00 0.3346456E+00, C2IN= 0.0000000E+00 0.0000000E+00
C1OUTS= 0.6586654E+00, C1INS= 0.0000000E+00, C2OUTS= 0.1312746E+00
C2INS= 0.0000000E+00, PT= 0.7899401E+00
R=   1, S=   1
XTME= 0.2661125E+00-0.1050917E-01, XTMES= 0.7092629E-01
CTME=-0.7338876E+00-0.1050917E-01, CTMES= 0.5387014E+00
R=   1, S=   2
XTME= 0.1272368E+00-0.9811844E-02, XTMES= 0.1628547E-01
CTME=-0.7727632E+00-0.9811844E-02, CTMES= 0.5972592E+00
R=   2, S=   1
XTME= 0.1298450E+00 0.6966247E-01, XTMES= 0.2171258E-01
CTME=-0.6701550E+00-0.3033753E-01, CTMES= 0.4500282E+00
XTMO= 0.4036774E-01-0.6299669E-02, XTMOS= 0.1669240E-02
CTMO=-0.6596323E+00-0.1562997E+00, CTMOS= 0.4595443E+00
R=   3, S=   1
XTME= 0.4398721E+00 0.7515975E-02, XTMES= 0.1935439E+00
CTME= 0.4398721E+00 0.7515975E-02, CTMES= 0.1935439E+00
XTMO=-0.2483288E-01 0.7582078E-01, XTMOS= 0.6365464E-02
CTMO=-0.2483288E-01 0.7582078E-01, CTMOS= 0.6365464E-02
R=   1, S=   1
XTEE=-0.7869720E-01 0.2917518E+00, XTEES= 0.9131238E-01
```

```
CTEE=-0.6786972E+00 0.9175183E-01, CTEES= 0.4690484E+00
R=  2, S=  1
XTEE=-0.9566223E-01 0.3063236E-01, XTEES= 0.1008861E-01
CTEE=-0.1956622E+00-0.7693676E+00, CTEES= 0.6302103E+00
XTEO= 0.1709717E+00 0.1320071E+00, XTEOS= 0.4665719E-01
CTEO=-0.2902830E-01-0.4679830E+00, CTEOS= 0.2196601E+00
R=  2, S=  2
XTEE=-0.6484716E-01 0.2201325E-01, XTEES= 0.4689738E-02
CTEE=-0.2148472E+00-0.6779867E+00, CTEES= 0.5058253E+00
XTEO= 0.3806341E-01 0.3481219E-01, XTEOS= 0.2660712E-02
CTEO=-0.4619366E+00-0.2651878E+00, CTEOS= 0.2837100E+00
R=  3, S=  1
XTEE=-0.7650289E-01 0.2868868E+00, XTEES= 0.8919273E-01
CTEE=-0.4765029E+00-0.1113133E+00, CTEES= 0.2394457E+00
XTEO=-0.1447939E+00 0.8617591E-01, XTEOS= 0.2839156E-01
CTEO=-0.4447939E+00-0.4138241E+00, CTEOS= 0.3690920E+00
R=  4, S=  1
XTEE= 0.2646753E+00-0.9678377E-01, XTEES= 0.7942011E-01
CTEE= 0.2646753E+00-0.9678377E-01, CTEES= 0.7942011E-01
XTEO=-0.2324401E+00-0.1543972E-01, XTEOS= 0.5426678E-01
CTEO=-0.2324401E+00-0.1543972E-01, CTEOS= 0.5426678E-01
R=  5, S=  1
XTEE=-0.2416435E-01 0.6750342E-01, XTEES= 0.5140628E-02
CTEE=-0.2416435E-01 0.6750342E-01, CTEES= 0.5140628E-02
XTEO=-0.2050299E+00 0.3951476E-01, XTEOS= 0.4359868E-01
CTEO=-0.2050299E+00 0.3951476E-01, CTEOS= 0.4359868E-01
PIN= 0.5935000E+01 PRM= 0.7659231E+00 PR= 0.5145061E+01
PTOTAL= 0.5935001E+01
PTA= 0.7899401E+00, PRMA= 0.7659230E+00
PHI2
 0.0000000E+00 0.3926991E-01 0.7853982E-01 0.1178097E+00 0.1570796E+00
 0.1963495E+00 0.2356195E+00 0.2748893E+00 0.3141593E+00 0.3534292E+00
 0.3926991E+00 0.4319690E+00 0.4712389E+00 0.5105088E+00 0.5497787E+00
 0.5890486E+00 0.6283185E+00 0.6675884E+00 0.7068583E+00 0.7461283E+00
 0.7853982E+00 0.8246680E+00 0.8639380E+00 0.9032079E+00 0.9424778E+00
 0.9817477E+00 0.1021018E+01 0.1060287E+01 0.1099557E+01 0.1138827E+01
 0.1178097E+01 0.1217367E+01 0.1256637E+01 0.1295907E+01 0.1335177E+01
 0.1374447E+01 0.1413717E+01 0.1452987E+01 0.1492257E+01 0.1531526E+01
 0.1570796E+01 0.1610066E+01 0.1649336E+01 0.1688606E+01 0.1727876E+01
 0.1767146E+01 0.1806416E+01 0.1845686E+01 0.1884956E+01 0.1924225E+01
 0.1963495E+01 0.2002765E+01 0.2042035E+01 0.2081305E+01 0.2120575E+01
 0.2159845E+01 0.2199115E+01 0.2238385E+01 0.2277655E+01 0.2316925E+01
 0.2356194E+01 0.2395464E+01 0.2434734E+01 0.2474004E+01 0.2513274E+01
 0.2552544E+01 0.2591814E+01 0.2631084E+01 0.2670354E+01 0.2709624E+01
```

0.2748893E+01 0.2788163E+01 0.2827433E+01 0.2866703E+01 0.2905973E+01
0.2945243E+01 0.2984513E+01 0.3023783E+01 0.3063053E+01 0.3102323E+01
0.3141593E+01

Z

0.0000000E+00 0.1570796E+00 0.3141593E+00 0.4712389E+00 0.6283185E+00
0.7853982E+00 0.9424778E+00 0.1099557E+01 0.1256637E+01 0.1413717E+01
0.1570796E+01 0.1727876E+01 0.1884956E+01 0.2042035E+01 0.2199115E+01
0.2356194E+01 0.2513274E+01 0.2670354E+01 0.2827433E+01 0.2984513E+01
0.3141593E+01

E3A1PS

0.1440311E+01 0.1437718E+01 0.1429980E+01 0.1417232E+01 0.1399688E+01
0.1377048E+01 0.1351482E+01 0.1321631E+01 0.1288592E+01 0.1252912E+01
0.1215171E+01 0.1175977E+01 0.1135949E+01 0.1095704E+01 0.1055840E+01
0.1016832E+01 0.9795074E+00 0.9440430E+00 0.9109506E+00 0.8805690E+00
0.8531612E+00 0.8289102E+00 0.8079215E+00 0.7902271E+00 0.7757931E+00
0.7645278E+00 0.7562906E+00 0.7509003E+00 0.7481426E+00 0.7477749E+00
0.7495310E+00 0.7531236E+00 0.7582478E+00 0.7645838E+00 0.7718006E+00
0.7795614E+00 0.7875286E+00 0.7953711E+00 0.8027719E+00 0.8094335E+00
0.8150869E+00 0.8194962E+00 0.8224644E+00 0.8238377E+00 0.8235077E+00
0.8214135E+00 0.8175403E+00 0.8119199E+00 0.8046260E+00 0.7957723E+00
0.7855061E+00 0.7740036E+00 0.7614635E+00 0.7480969E+00 0.7341363E+00
0.7197976E+00 0.7053063E+00 0.6908690E+00 0.6766868E+00 0.6629317E+00
0.6497573E+00 0.6372907E+00 0.6256334E+00 0.6148599E+00 0.6050196E+00
0.5961373E+00 0.5882164E+00 0.5812401E+00 0.5751751E+00 0.5699735E+00
0.5655764E+00 0.5619160E+00 0.5589190E+00 0.5565088E+00 0.5546093E+00
0.5531470E+00 0.5520541E+00 0.5512708E+00 0.5507482E+00 0.5504497E+00
0.5503528E+00

E3A1ZS

0.1272854E+01 0.1260026E+01 0.1223450E+01 0.1168599E+01 0.1103745E+01
0.1038591E+01 0.9823436E+00 0.9416218E+00 0.9192008E+00 0.9145649E+00
0.9260527E+00 0.9529779E+00 0.9961452E+00 0.1056472E+01 0.1132810E+01
0.1220551E+01 0.1311808E+01 0.1396856E+01 0.1465959E+01 0.1510994E+01
0.1526623E+01

E3A2PS

0.9903874E-01 0.9845793E-01 0.9672385E-01 0.9386183E-01 0.8991543E-01
0.8495012E-01 0.7906191E-01 0.7239684E-01 0.6519208E-01 0.5785974E-01
0.5114439E-01 0.4634207E-01 0.4529315E-01 0.4951921E-01 0.5909353E-01
0.7301147E-01 0.9023315E-01 0.1100420E+00 0.1319709E+00 0.1556828E+00
0.1808922E+00 0.2073220E+00 0.2346819E+00 0.2626592E+00 0.2909158E+00
0.3190908E+00 0.3468060E+00 0.3736736E+00 0.3993057E+00 0.4233238E+00
0.4453706E+00 0.4651202E+00 0.4822889E+00 0.4966459E+00 0.5080220E+00
0.5163180E+00 0.5215109E+00 0.5236595E+00 0.5229069E+00 0.5194818E+00
0.5136979E+00 0.5059494E+00 0.4967074E+00 0.4865116E+00 0.4759608E+00
0.4657024E+00 0.4564196E+00 0.4488171E+00 0.4436061E+00 0.4414887E+00

```
 0.4431412E+00 0.4491977E+00 0.4602347E+00 0.4767545E+00 0.4981731E+00
 0.5278062E+00 0.5628593E+00 0.6044197E+00 0.6524488E+00 0.7067835E+00
 0.7671261E+00 0.8330511E+00 0.9040133E+00 0.9793465E+00 0.1058277E+01
 0.1139834E+01 0.1223366E+01 0.1307552E+01 0.1391426E+01 0.1473883E+01
 0.1553818E+01 0.1630129E+01 0.1701743E+01 0.1767639E+01 0.1826864E+01
 0.1878551E+01 0.1921937E+01 0.1956376E+01 0.1981352E+01 0.1996489E+01
 0.2001561E+01
E3A22S
 0.8881345E+00 0.8736060E+00 0.8332719E+00 0.7765757E+00 0.7178044E+00
 0.6724335E+00 0.6503232E+00 0.6483415E+00 0.6563746E+00 0.6552080E+00
 0.6338391E+00 0.5882396E+00 0.5245103E+00 0.4614712E+00 0.4306867E+00
 0.4585061E+00 0.5370481E+00 0.6347617E+00 0.7230313E+00 0.7829823E+00
 0.8040966E+00
BKAPLT
 0.5600000E+01
PINC
 0.5935000E+01
PTRAN
 0.7899401E+00
PREFL
 0.5145061E+01
```

Discussion of the above output data

First, the input data read in by the first group of statements listed in Section 2.1.1 are written out. Then, the variable NMAX and the array MM are written out after being calculated in the subroutine MODES (see Chapter 4 of [3]). Next, the number KTM of waveguide modes used in each rectangular waveguide, the number KTE of TE waveguide modes used in each rectangular waveguide, K1 defined by

$$\mathrm{K1} = \mathrm{KTM} + \mathrm{KTE} \tag{2.59}$$

and BKB defined by

$$\mathrm{BKB} = kb \tag{2.60}$$

are written out. The elements $\{-j\eta Y_{ii}^{\mathrm{rec}},\ i = 1, 2, \cdots, K2\}$, which will appear in (4.12) and (4.17), are written under the heading YREC. Here, Y_{ii}^{rec} is the ii^{th} element of $Y^1 + Y^2$ where Y^1 and Y^2 are the rectangular waveguide admittance matrices given by eqs. (2.25) and (2.27) of [1].

Next, the input data read in by the second group of statements listed in Section 2.1.1 are written out. Then, RTM of (7.29), RTE of (7.47), {STM(R), R = 1, 2, $\cdots$, RTM} of (7.30) and {STE(R), R = 1, 2, $\cdots$, RTE} of (7.48) are written out. The values of RTM, RTE, STM, and STE are, respectively, those of RTMR, RTER, STMR, and STER for which the ranges of values of r and s in (2.25) and (2.26) cover all the propagating but no evanescent modes in the circular waveguide.

The normalized elements $\{-j\sqrt{\bar{\eta}}\, I_i^{\alpha\beta}\}$ of the excitation vector given by (3.13) are written out under the heading TI. The elements $\{I_i^{\alpha\beta}\}$ are, as can be gleaned from Section 7.4.4, ordered as

$$\begin{aligned} &\{I_i^{1\mathrm{TM}}, i = 1, 2, \cdots, \mathrm{KTM}\}, \{I_i^{1\mathrm{TE}}, i = 1, 2, \cdots, \mathrm{KTE}\} \\ &\{I_i^{2\mathrm{TM}}, i = 1, 2, \cdots, \mathrm{KTM}\}, \{I_i^{2\mathrm{TE}}, i = 1, 2, \cdots, \mathrm{KTE}\}. \end{aligned} \tag{2.61}$$

The elements of $\vec{V}/\sqrt{\bar{\eta}}$ where $\vec{V}$ is the column vector that satisfies the matrix equation (7.11) are printed under the heading V. The elements of $\vec{V}$ are ordered as the elements $\{I_i^{\alpha\beta}\}$ of the excitation vector are ordered:

$$\begin{aligned} &\{V_i^{1\mathrm{TM}}, i = 1, 2, \cdots, \mathrm{KTM}\}, \{V_i^{1\mathrm{TE}}, i = 1, 2, \cdots, \mathrm{KTE}\} \\ &\{V_i^{2\mathrm{TM}}, i = 1, 2, \cdots, \mathrm{KTM}\}, \{V_i^{2\mathrm{TE}}, i = 1, 2, \cdots, \mathrm{KTE}\}. \end{aligned} \tag{2.62}$$

The written values of C1OUT, C1IN, C2OUT, C2IN, C1OUTS, C1INS, C2OUTS, C2INS, and PT are described in Section 2.2.1.

The 16 lines following the line where PT is written contain the normalized coefficients of the TM propagating modes in the circular waveguide. As indicated in Section 2.2.1, CTME and CTMO are, respectively, the normalized coefficients of the reflected waves of the even and odd $\mathrm{TM}_{\mathrm{R-1,S}}$ propagating modes that exist in the circular waveguide of Fig. 2. Also, CTMES and CTMOS are the squares of the magnitudes of CTME and CTMO, respectively. Other quantities written in the 16 lines cited in the first sentence of this paragraph are XTME of (7.146), XTMO of (7.152), XTMES of (7.150), and XTMOS of (7.156).

Now, XTME and XTMO are, respectively, the normalized coefficients of the waves of the even and odd $\mathrm{TM}_{\mathrm{R-1,S}}$ propagating modes radiated by the combination of the magnetic currents $-\underline{M}^{(1)}$ and $-\underline{M}^{(2)}$ in the circular waveguide of Fig. 2. In other words, XTME is the contribution to CTME

due to the magnetic currents in the circular waveguide, and XTMO is what the magnetic currents contribute to CTMO. Also, XTMES and XTMOS are the squares of the magnitudes of XTME and XTMO, respectively.

The next 44 lines contain the normalized coefficients of the TE propagating modes in the circular waveguide. As indicated in Section 2.2.1, CTEE and CTEO are, respectively, the normalized coefficients of the reflected waves of the even and odd $TE_{R-1,S}$ propagating modes that exist in the circular waveguide of Fig. 2. Also, CTEES and CTEOS are the squares of the magnitudes of CTEE and CTEO, respectively. Other quantities written in the 44 lines cited in the first sentence of this paragraph are XTEE of (7.186), XTEO of (7.192), XTEES of (7.190) and XTEOS of (7.196).

Now XTEE and XTEO are, respectively, the normalized coefficients of the waves of the even and odd $TE_{R-1,S}$ propagating modes radiated by the combination of the magnetic currents $-\underline{M}^{(1)}$ and $-\underline{M}^{(2)}$ in the circular waveguide of Fig. 2. In other words, XTEE is the contribution to CTEE due to the magnetic currents in the circular waveguide, and XTEO is what the magnetic currents contribute to CTEO. Also, XTEES and XTEOS are the squares of the magnitudes of XTEE and XTEO, respectively.

The next three lines show the values of PIN, PRM, PR, PTOTAL, PTA, and PRMA. Here, PIN is the time-average power P_{inc} of (5.44) of the z-traveling (incident) waves in the circular waveguide, PRM is the time-average power P_{rm} of (5.34) radiated by the magnetic currents in the circular waveguide, and PR is the time-average power P_r of (5.45) reflected in the circular waveguide. Also, PTOTAL is the sum of P_r and the time-average power P_t transmitted into the rectangular waveguides. Here, P_t is given by (2.52) in which $P_t^{(1)}$ and $P_t^{(2)}$ are given by (4.7) and (4.8). The variable PTA is P_{ta} where P_{ta} is an alternative expression for the time-average power transmitted into the rectangular waveguides:

$$P_{ta} = P_{ta}^{(1)} + P_{ta}^{(2)} \tag{2.63}$$

where $P_{ta}^{(1)}$ and $P_{ta}^{(2)}$ are given by (4.12) and (4.17), respectively. The variable PRMA is P_{rma} where P_{rma} is the alternative expression (5.43) for the time-average power radiated by the magnetic currents in the circular waveguide.

The computer program variable PT described in Section 2.2.1 and PIN, PRM, PTOTAL, PTA, and PRMA cited in the first sentence of the previous

paragraph should satisfy

$$\mathrm{PT} = \mathrm{PTA} \tag{2.64}$$

$$\mathrm{PIN} = \mathrm{PTOTAL} \tag{2.65}$$

$$\mathrm{PRM} = \mathrm{PRMA}. \tag{2.66}$$

The numerical values of these computer program variables in the exemplary output data satisfy the above equations within roundoff error.

The J^{th} of the NPHI entries written under the heading PHI2 is $\phi_{\mathrm{J}}^{(2)}$ given by

$$\phi_{\mathrm{J}}^{(2)} = \frac{(\mathrm{J}-1)\pi}{\mathrm{NPHI}-1}. \tag{2.67}$$

The J^{th} of the $\mathrm{NZ}^{\mathrm{th}}$ entries written under the heading Z is $z_{\mathrm{J}}^{(1)}$ given by

$$z_{\mathrm{J}}^{(1)} = \frac{(\mathrm{J}-1)\pi}{\mathrm{NZ}-1}. \tag{2.68}$$

The quantities $\{\phi_{\mathrm{J}}^{(2)},\ \mathrm{J} = 1, 2, \cdots, \mathrm{NPHI}\}$ are useful abscissas for plotting the ordinates written under the headings E3A1PS and E3A2PS. The quantities $\{z_{\mathrm{J}}^{(1)},\ \mathrm{J} = 1, 2, \cdots, \mathrm{NZ}\}$ are useful abscissas for plotting the ordinates written under the headings E3A1ZS and E3A2ZS.

The J^{th} of the NPHI entries written under the heading E3A1PS is E3A1PS(J) of (2.54), and the J^{th} of the NZ entries written under the heading E3A1ZS is E3A1ZS(J) of (2.55). The J^{th} of the NPHI entries written under the heading E3A2PS is E3A2PS(J) of (2.56), and the J^{th} of the NZ entries written under the heading E3A2ZS is E3A2ZS(J) of (2.57).

The I^{th} of the KAM entries written under the heading BKAPLT is ka given by the right-hand side of (2.15) with KA replaced by I:

$$ka = \mathrm{BKA0} + (\mathrm{I}-1) * \mathrm{DBKA} \tag{2.69}$$

The I^{th} of the KAM entries written under the heading PINC is the value of PIN that was calculated with ka given by (2.69). The I^{th} of the KAM entries written under the heading PTRAN is the value of PT that was calculated with ka given by (2.69). The I^{th} of the KAM entries written under the heading PREFL is the value of PR that was calculated with ka given by (2.69).

Output data written in the file BESOUT.DAT by statements in the subroutine BESIN

These output data are the same as those described in Section 2.2.2 of [3].

2.3 Minimum Allocations

The minimum storage space that must be allocated to some arrays in the computer program depends on the values of the input variables B, C, BKM, KAM, KE3M, NPHI, NZ, RTMR, RTER, (STMR(R), R = 1, RTMR), and (STER(R), R = 1, RTER). There is also dependence on the maximum of the values of ka in (2.15). Furthermore, dependence on XM arises when XM is extraordinarily large (see the last two paragraphs of this section).

Minimum allocations in the main program are given by

```
MM(NMAX),BMN(KTE),BMN2(KTE),PH1(3*PMAX),PH2(3*PMAX),
PH3(3*PMAX),PH4(3*PMAX),D3(NMAX),G4(NMAX),DTM(NMAX),DTE(NMAX),
E3A1P(NPHI),E3A1Z(NZ),E3A2P(NPHI),E3A2Z(NZ),Y(K2*K2),TI(K2),
V(K2),YREC(K2),GTM(NMAX),GTE(NMAX),TMP(NMAX),TMM(NMAX),
TEP(NMAX),TEM(NMAX),DQTM(NMAX),DQTE(NMAX),PHI1(NPHI),
PHI2(NPHI),Z(NZ),PTRAN(KAM),PREFL(KAM),BKAPLT(KAM),SINP(PMAX),
SINQ(PMAX),E3A1PS(NPHI),E3A2PS(NPHI),E3A1ZS(NZ),E3A2ZS(NZ),
IPS(K2),KE3(KE3M),STM(RTM),STE(RTE),STMR(RTMR),STER(RTER),
ITME(KTE*NTME),ITMO(KTE*NTMO),ITEEP(KTE*NTEE),ITEEZ(KTE*NTEE),
ITEOP(KTE*NTEO),ITEOZ(KTE*NTEO),PINC(KAM),BTM(STMM,RTM),
BTE(STEM,RTE),BTME(MAX(STMM,STMR),MAX(RTM,RTMR)),
BTMO(MAX(STMM,STMR),MAX(RTM,RTMR)),
BTEE(MAX(STEM,STER),MAX(RTE,RTER)),
BTEO(MAX(STEM,STER),MAX(RTE,RTER))
```

where "MAX" denotes the maximum value of the quantities that follow in parentheses. In the above list of minimum allocations,

$$\mathrm{NMAX} = 1 + \left\{ \begin{array}{c} \text{the maximum value of } n \text{ such that} \\ (n\pi)\,(\mathrm{B/C}) \le \mathrm{BKM} \end{array} \right\} \tag{2.70}$$

and

$$\mathrm{KTE} = -1 + \left\{ \begin{array}{c} \text{the number of combinations of nonnegative} \\ \text{integers } m \text{ and } n \text{ such that} \\ \sqrt{(m\pi)^2 + ((n\pi)\mathrm{B/C})^2} \le \mathrm{BKM} \end{array} \right\}. \tag{2.71}$$

Furthermore,

$$\text{PMAX} = 1 + \left\{ \begin{array}{c} \text{the maximum value of } p \text{ such that} \\ p\pi \leq \text{BKM} \end{array} \right\} \tag{2.72}$$

and

$$\text{K2} = 2 * (\text{KTM} + \text{KTE}) \tag{2.73}$$

where KTE is given by (2.71) and

$$\text{KTM} = \left\{ \begin{array}{c} \text{the number of combinations of positive} \\ \text{integers } m \text{ and } n \text{ such that} \\ \sqrt{(m\pi)^2 + ((n\pi)(\text{B}/\text{C}))^2} \leq \text{BKM} \end{array} \right\}. \tag{2.74}$$

Moreover,

$$\text{NTME} = \left\{ \begin{array}{c} \text{the number of positive roots } x_{rs} \text{ of} \\ \{J_r,\ r = 0, 1, 2, \cdots\} \text{ such that } x_{rs} < ka \end{array} \right\} \tag{2.75}$$

$$\text{NTMO} = \left\{ \begin{array}{c} \text{the number of positive roots } x_{rs} \text{ of} \\ \{J_r,\ r = 1, 2, \cdots\} \text{ such that } x_{rs} < ka \end{array} \right\} \tag{2.76}$$

$$\text{NTEE} = \left\{ \begin{array}{c} \text{the number of positive roots } x'_{rs} \text{ of} \\ \{J'_r,\ r = 0, 1, 2, \cdots\} \text{ such that } x'_{rs} < ka \end{array} \right\} \tag{2.77}$$

$$\text{NTEO} = \left\{ \begin{array}{c} \text{the number of positive roots } x'_{rs} \text{ of} \\ \{J'_r,\ r = 1, 2, \cdots\} \text{ such that } x'_{rs} < ka \end{array} \right\} \tag{2.78}$$

where ka is the maximum value of the right-hand side of (2.15):

$$ka = \left\{ \begin{array}{ll} \text{BKA0}, & \text{DBKA} \leq 0 \\ \text{BKA0} + (\text{KAM} - 1) * \text{DBKA}, & \text{DBKA} > 0. \end{array} \right\} \tag{2.79}$$

In (2.75) and (2.76), J_r is the Bessel function of the first kind of order r, and its positive roots $\{x_{rs},\ s = 1, 2, \cdots\}$ are ordered such that

$$x_{r1} < x_{r2} < x_{r3} \cdots . \tag{2.80}$$

In (2.77) and (2.78), J'_r is the derivative of J_r with respect to its argument and the positive roots $\{x'_{rs}, s = 1, 2, \cdots\}$ of J'_r are ordered such that

$$x'_{r1} < x'_{r2} < x'_{r3} \cdots . \tag{2.81}$$

Other variables appearing in the minimum allocations presented in the previous paragraph are

$$\text{RTM} = 1 + (\text{the maximum value of } r \text{ in } (2.75)) \tag{2.82}$$

$$\text{STMM} = \text{the maximum value of } s \text{ in } (2.75) \tag{2.83}$$

$$\text{RTE} = 1 + (\text{the maximum value of } r \text{ in } (2.77)) \tag{2.84}$$

$$\text{STEM} = \text{the maximum value of } s \text{ in } (2.77) . \tag{2.85}$$

If there are no roots in (2.75), then RTM = 1 and STMM = 0. There is at least one root in (2.77); the root x'_{11} has to be allowed. If it were not allowed, then there would be no propagating mode with which to excite the circular waveguide.

Now, NMAX of (2.70), KTE of (2.71), PMAX of (2.72), K2 of (2.73), KTM of (2.74), RTM of (2.82), and RTE of (2.84) are variables in the main program, but NTME of (2.75), NTMO of (2.76), NTEE of (2.77), NTEO of (2.78), STMM of (2.83), and STEM of (2.85) are not. All 13 variables mentioned in the previous sentence are meant to be integer variables. As defined by (2.71), KTE is the number of TE modes retained in the expansion of the field in each rectangular waveguide. Furthermore, KTM of (2.74) is the number of TM modes retained in the expansion of the field in each rectangular waveguide. The quantities NTME of (2.75), NTMO of (2.76), NTEE of (2.77), and NTEO of (2.78) are, respectively, the numbers of even TM, odd TM, even TE, and odd TE modes that propagate in the circular waveguide.

One who is not inclined to calculate the precise values of NMAX, KTE, PMAX, K2, KTM, NTME, NTMO, NTEE, NTEO, RTM, STMM, RTE, and STEM from their definitions is advised to use educated guesses for these variables in order to specify allocations. If one can get the program to run to completion without any error messages, then he can use the output data to easily obtain the precise values of the variables mentioned in the previous sentence. The actual values of NMAX, KTE, KTM, RTM, and RTE appear in these output data. The first integer number written under the heading MM

is PMAX. The value of K2 is twice that of the printed value of K1. Moreover, NTME is the sum of the integer numbers written under the heading STM, NTMO is this sum minus the first integer number written under the heading STM, NTEE is the sum of the integer numbers written under the heading STE, and NTEO is this sum minus the first integer number written under the heading STE. Moreover, STMM is the largest of the integer numbers written under the heading STM, and STEM is the largest of the integer numbers written under the heading STE.

Minimum allocations in the subroutines are given by

$$\left\{ \begin{array}{c} \text{MM(NMAX),BMN(KTE),} \\ \text{BMN2(KTE)} \end{array} \right\} \text{in the subroutine MODES,}$$

$$\left\{ \begin{array}{c} \text{PH1(3} * \text{PMAX),PH2(3} * \text{PMAX),} \\ \text{PH3(3} * \text{PMAX),PH4(3} * \text{PMAX)} \end{array} \right\} \text{in the subroutine PHI}$$

$$\left\{ \begin{array}{c} \text{D3(NMAX),G4(NMAX),D(NMAX),} \\ \text{CP(NMAX),CM(NMAX),} \\ \text{DQ(NMAX),G(NMAX)} \end{array} \right\} \text{in the subroutine DGN,}$$

$$\left\{ \text{UL(K2} * \text{K2),SCL(K2),IPS(K2)} \right\} \text{in the subroutine DECOMP, and}$$

$$\left\{ \text{UL(K2} * \text{K2),B(K2),X(K2),IPS(K2)} \right\} \text{in the subroutine SOLVE.}$$

A blank or labeled common block that is used in two or more program segments must be defined exactly the same in each of these program segments.† Therefore, any dimensioned variable in a common block that is used in two or more program segments must have the same allocation in each of these program segments. That is why each of the arrays PH1, PH2, PH3, and PH4 in the subroutine PHI was allocated 3*PMAX entries instead of 2*PMAX entries.

If XM is so large that

$$x'_{1,200} \leq \text{XM} \tag{2.86}$$

where $x'_{1,200}$ is the 200^{th} root of J'_1, then the "200" in the fifth statement in DO loop 19 of the main program must be replaced by I where I is an integer

†Here, a program segment is either the main program of one of the subprograms.

at least so large that[†]

$$x'_{1,I} > \mathrm{XM}. \tag{2.87}$$

Also, the maximum value of the index S of DO loop 25 in the subroutine BESIN must be increased from 200 to I. Furthermore, the maximum value of the indices S of DO loops 14 and 15 in the subroutine BES must be increased from 200 to I. Accompanying minimum allocations are given by

$$\begin{array}{rl} \{\mathrm{XJ(I),XJP(I)}\} & \text{in the main program,} \\ \{\mathrm{A(I),AP(I)}\} & \text{in the subroutine BESIN,} \\ \left\{\begin{array}{c}\mathrm{A(I),AP(I),} \\ \mathrm{XJ(I),XJP(I)}\end{array}\right\} & \text{in the subroutine BES,} \\ \{\mathrm{A(I),AP(I)}\} & \text{in the subroutine INTERPOL, and} \\ \{\mathrm{X(I)}\} & \text{in the subroutine DGN.} \end{array}$$

If XM is so large that

$$x'_{500,1} \leq \mathrm{XM} \tag{2.88}$$

where $x'_{500,1}$ is the first root of J'_{500}, then the upper limit on the index of DO loop 19 in the main program must be increased from 500 to J where J is an integer at least so large that

$$x'_{J,1} > \mathrm{XM}. \tag{2.89}$$

[†]See the last paragraph of Section 2.3 of [3]. Note that the first three I's in the curly brackets in the line after eq. (2.73) of [3] are erroneous. Any integer variable other than I is meant here.

Chapter 3

The Excitation Vector

The waveguide mode converter is excited by a mixture of the z-traveling waves of the propagating modes in the circular waveguide. The elements of the excitation vector are obtained by substituting the magnetic field of these waves and their reflections about the short at $z = L_3$ into eq. (2.24)† of [1]. This magnetic field is that which would exist in the circular waveguide if both apertures were shorted.

3.1 The Electromagnetic Field of the Mixture of z-Traveling Waves in the Circular Waveguide

The electric and magnetic fields of the mixture of z-traveling waves in the circular waveguide are $\underline{E}^{(3+)}(\underline{J}^{imp}, \underline{\Omega})$ and $\underline{H}^{(3+)}(\underline{J}^{imp}, \underline{\Omega})$ given by

$$\underline{E}^{(3+)}(\underline{J}^{imp}, \underline{\Omega}) = \sum_{r,s} \sum_{f=e,o} \left(\frac{B_{rs}^{TMf} e^{j\beta_{rs}^{TM} L_3}}{\sqrt{Z_{rs}^{TMeo}}} \right) \underline{E}_{rs}^{TMf+}$$

†An equation that appears in a reference is cited by placing "eq." before the equation number. An equation that appears in the present report is cited by writing only its number in parentheses. However, an equation number at the beginning of a sentence is always preceded by "Equation".

$$+\sum_{r,s}\sum_{f=e,o}\left(\frac{B_{rs}^{\mathrm{TE}f}e^{j\beta_{rs}^{\mathrm{TE}}L_3}}{\sqrt{Y_{rs}^{\mathrm{TE}eo}}}\right)\underline{E}_{rs}^{\mathrm{TE}f+} \tag{3.1}$$

$$\underline{H}^{(3+)}(\underline{J}^{\mathrm{imp}},\underline{0}) = \sum_{r,s}\sum_{f=e,o}\left(\frac{B_{rs}^{\mathrm{TM}f}e^{j\beta_{rs}^{\mathrm{TM}}L_3}}{\sqrt{Z_{rs}^{\mathrm{TM}eo}}}\right)\underline{H}_{rs}^{\mathrm{TM}f+}$$

$$+\sum_{r,s}\sum_{f=e,o}\left(\frac{B_{rs}^{\mathrm{TE}f}e^{j\beta_{rs}^{\mathrm{TE}}L_3}}{\sqrt{Y_{rs}^{\mathrm{TE}eo}}}\right)\underline{H}_{rs}^{\mathrm{TE}f+} \tag{3.2}$$

where the B's are constants, $(\underline{E}_{rs}^{\mathrm{TM}e+},\underline{H}_{rs}^{\mathrm{TM}e+})$ is given by eq. (B.1) of [1], $(\underline{E}_{rs}^{\mathrm{TM}o+},\underline{H}_{rs}^{\mathrm{TM}o+})$ is given by eq. (B.26) of [1], $(\underline{E}_{rs}^{\mathrm{TE}e+},\underline{H}_{rs}^{\mathrm{TE}e+})$ is given by eq. (B.35) of [1], and $(\underline{E}_{rs}^{\mathrm{TE}o+},\underline{H}_{rs}^{\mathrm{TE}o+})$ is given by eq. (B.55) of [1]. The first summation with respect to the indices r and s on the right-hand side of each of (3.1) and (3.2) is taken over all the propagating TM modes of the circular waveguide. The second summation with respect to the indices r and s on the right-hand side of each of (3.1) and (3.2) is taken over all the propagating TE modes of the circular waveguide.

The quantity that $B_{rs}^{\mathrm{TM}f}$ multiplies on the right-hand side of (3.2) reduces to the magnetic field $\underline{h}_{rs}^{\mathrm{TM}f}/\sqrt{Z_{rs}^{\mathrm{TM}eo}}$ when $z = L_3$. This magnetic field is that of a TM wave whose time-average power is unity. The quantity that $B_{rs}^{\mathrm{TE}f}$ multiplies on the right-hand side of (3.1) reduces to the electric field $\underline{e}_{rs}^{\mathrm{TE}f}/\sqrt{Y_{rs}^{\mathrm{TE}eo}}$ when $z = L_3$. This electric field is that of a TE wave whose time-average power is unity. Here, $\underline{e}_{rs}^{\mathrm{TM}e}$, $\underline{h}_{rs}^{\mathrm{TM}e}$, $\underline{e}_{rs}^{\mathrm{TM}o}$, $\underline{h}_{rs}^{\mathrm{TM}o}$, $\underline{e}_{rs}^{\mathrm{TE}e}$, $\underline{h}_{rs}^{\mathrm{TE}e}$, $\underline{e}_{rs}^{\mathrm{TE}o}$, and $\underline{h}_{rs}^{\mathrm{TE}o}$ are given, respectively, by eqs. (B.22), (B.23), (B.33), (B.34), (B.51), (B.52), (B.62), and (B.63) of [1]. Moreover, $Z_{rs}^{\mathrm{TM}eo}$ and $Y_{rs}^{\mathrm{TE}eo}$ are given, respectively, by eqs. (B.25) and (B.54) of [1] where

$$\gamma_{rs}^{\mathrm{TM}} = j\beta_{rs}^{\mathrm{TM}} = j\sqrt{k^2-(k_{rs}^{\mathrm{TM}})^2} \tag{3.3}$$

$$\gamma_{rs}^{\mathrm{TE}} = j\beta_{rs}^{\mathrm{TE}} = j\sqrt{k^2-(k_{rs}^{\mathrm{TE}})^2}\,. \tag{3.4}$$

In (3.3) and (3.4), k is the wavenumber and

$$k_{rs}^{\mathrm{TM}} = \frac{x_{rs}}{a} \tag{3.5}$$

$$k_{rs}^{\mathrm{TE}} = \frac{x'_{rs}}{a} \tag{3.6}$$

where x_{rs} is the s^{th} root of the Bessel function J_r and x'_{rs} is the s^{th} root of J'_r. Here, J'_r is the derivative of J_r with respect to its argument.

3.2 The Electromagnetic Field That Would Exist in the Circular Waveguide if Both Apertures Were Shorted

If the apertures A_1 and A_2 in Fig. 1 were closed with perfect conductors, then the electromagnetic field $(\underline{E}^{(3)}(\underline{J}^{\text{imp}},\underline{0}), \underline{H}^{(3)}(\underline{J}^{\text{imp}},\underline{0}))$ in the circular waveguide would be the z-traveling waves on the right-hand sides of (3.1) and (3.2) plus their reflections about $z = L_3$. Adding these reflections to the right-hand sides of (3.1) and (3.2), we obtain

$$\underline{E}^{(3)}(\underline{J}^{\text{imp}},\underline{0}) = \sum_{r,s}\sum_{f=e,o}\left(\frac{B_{rs}^{\text{TM}f}e^{j\beta_{rs}^{\text{TM}}L_3}}{\sqrt{Z_{rs}^{\text{TM}eo}}}\right)\left(\underline{E}_{rs}^{\text{TM}f+} + e^{-2j\beta_{rs}^{\text{TM}}L_3}\underline{E}_{rs}^{\text{TM}f-}\right)$$
$$+\sum_{r,s}\sum_{f=e,o}\left(\frac{B_{rs}^{\text{TE}f}e^{j\beta_{rs}^{\text{TE}}L_3}}{\sqrt{Y_{rs}^{\text{TE}eo}}}\right)\left(\underline{E}_{rs}^{\text{TE}f+} - e^{-2j\beta_{rs}^{\text{TE}}L_3}\underline{E}_{rs}^{\text{TE}f-}\right) \quad (3.7)$$

$$\underline{H}^{(3)}(\underline{J}^{\text{imp}},\underline{0}) = \sum_{r,s}\sum_{f=e,o}\left(\frac{B_{rs}^{\text{TM}f}e^{j\beta_{rs}^{\text{TM}}L_3}}{\sqrt{Z_{rs}^{\text{TM}eo}}}\right)\left(\underline{H}_{rs}^{\text{TM}f+} + e^{-2j\beta_{rs}^{\text{TM}}L_3}\underline{H}_{rs}^{\text{TM}f-}\right)$$
$$+\sum_{r,s}\sum_{f=e,o}\left(\frac{B_{rs}^{\text{TE}f}e^{j\beta_{rs}^{\text{TE}}L_3}}{\sqrt{Y_{rs}^{\text{TE}eo}}}\right)\left(\underline{H}_{rs}^{\text{TE}f+} - e^{-2j\beta_{rs}^{\text{TE}}L_3}\underline{H}_{rs}^{\text{TE}f-}\right) \quad (3.8)$$

where $(\underline{E}_{rs}^{\text{TM}e-}, \underline{H}_{rs}^{\text{TM}e-})$, $(\underline{E}_{rs}^{\text{TM}o-}, \underline{H}_{rs}^{\text{TM}o-})$, $(\underline{E}_{rs}^{\text{TE}e-}, \underline{H}_{rs}^{\text{TE}e-})$, and $(\underline{E}_{rs}^{\text{TE}o-}, \underline{H}_{rs}^{\text{TE}o-})$ are given, respectively, by eqs. (B.2), (B.27), (B.36), and (B.56) of [1]. Obviously, $\underline{E}^{(3)}(\underline{J}^{\text{imp}},\underline{0})$ and $\underline{H}^{(3)}(\underline{J}^{\text{imp}},\underline{0})$ of (3.7) and (3.8) are propagating mode contributions to the total electromagnetic field in the cylindrical waveguide. In Chapter 4, (3.7) will be used in obtaining expressions for the coefficients of the propagating modes of the total electromagnetic field in the cylindrical waveguide.

3.3 The Excitation Vector in Terms of the B's in (3.1)

The excitation vector is the column vector on the right-hand side of eq. (2.22) of [1] where the i^{th} element of $\bar{I}^{\alpha\beta}$ is $I_i^{\alpha\beta}$ given by eq. (2.24) of [1]:

$$I_i^{\alpha\beta} = -\iint_{A_a} \underline{M}_{mn}^{\alpha\beta} \cdot \underline{H}^{(3)}(\underline{J}^{imp}, \underline{0})\, ds \tag{3.9}$$

where i is related to m and n. The relationship between i and m and n is the same as in eq. (2.24) of [1]. Substituting eqs. (B.1), (B.2), (B.26), (B.27), (B.35), (B.36), (B.55), and (B.56) of [1] for $\underline{H}_{rs}^{\mathrm{TMe}+}$, $\underline{H}_{rs}^{\mathrm{TMe}-}$, $\underline{H}_{rs}^{\mathrm{TMo}+}$, $\underline{H}_{rs}^{\mathrm{TMo}-}$, $\underline{H}_{rs}^{\mathrm{TEe}+}$, $\underline{H}_{rs}^{\mathrm{TEe}-}$, $\underline{H}_{rs}^{\mathrm{TEo}+}$, and $\underline{H}_{rs}^{\mathrm{TEo}-}$ in (3.8), we obtain

$$\underline{H}^{(3)}(\underline{J}^{imp}, \underline{0}) = \sum_{r,s} \sum_{f=e,o} B_{rs}^{\mathrm{TM}f} \underline{H}_{rs}^{\mathrm{TM}f} + \sum_{r,s} \sum_{f=e,o} B_{rs}^{\mathrm{TE}f} \underline{H}_{rs}^{\mathrm{TE}f} \tag{3.10}$$

where

$$\underline{H}_{rs}^{\mathrm{TM}f} = \frac{2}{\sqrt{Z_{rs}^{\mathrm{TMeo}}}} \underline{h}_{rs}^{\mathrm{TM}f} \cos(\beta_{rs}^{\mathrm{TM}}(L_3 - z)) \tag{3.11}$$

$$\begin{aligned} \underline{H}_{rs}^{\mathrm{TE}f} = \frac{2}{\sqrt{Y_{rs}^{\mathrm{TEeo}}}} \Big\{ & Y_{rs}^{\mathrm{TEeo}} \underline{h}_{rs}^{\mathrm{TE}f} \cos(\beta_{rs}^{\mathrm{TE}}(L_3 - z)) \\ & + \underline{u}_z \frac{(k_{rs}^{\mathrm{TE}})^2 \psi_{rs}^{\mathrm{TE}f}}{\omega\mu} \sin(\beta_{rs}^{\mathrm{TE}}(L_3 - z)) \Big\}. \end{aligned} \tag{3.12}$$

Substitution of (3.10) into (3.9) gives, upon multiplication by $-j\sqrt{\eta}$,

$$\begin{aligned} -j\sqrt{\eta} I_i^{\alpha\beta} = -j \sum_{r,s} \Big(& B_{rs}^{\mathrm{TMe}} I_i^{\alpha\beta,\mathrm{TMe}} + B_{rs}^{\mathrm{TMo}} I_i^{\alpha\beta,\mathrm{TMo}} \\ & + B_{rs}^{\mathrm{TEe}} I_i^{\alpha\beta,\mathrm{TEe}} + B_{rs}^{\mathrm{TEo}} I_i^{\alpha\beta,\mathrm{TEo}} \Big) \end{aligned} \tag{3.13}$$

where

$$I_i^{\alpha\beta,\delta f} = -\sqrt{\eta} \iint_{A_a} \underline{M}_{mn}^{\alpha\beta} \cdot \left[\underline{H}_{rs}^{\delta f}\right]_{\rho=a} ds, \quad \begin{cases} \alpha = 1,2 \\ \beta = \mathrm{TM,TE} \\ \delta = \mathrm{TM,TE} \\ f = e,o. \end{cases} \tag{3.14}$$

Here, $\eta = \sqrt{\mu/\epsilon}$ is the intrinsic impedance. In (3.14), $\left[\underline{H}^{\delta f}_{rs}\right]_{\rho=a}$ is $\underline{H}^{\delta f}_{rs}$ evaluated at $\rho = a$. Suitable expressions for $\underline{M}^{\alpha\beta}_{mn}$ and $\left[\underline{H}^{\delta f}_{rs}\right]_{\rho=a}$ must be found before the integral on the right-hand side of (3.14) can be evaluated.

3.4 Evaluation of Expression (3.14)

3.4.1 The $\underline{M}$'s in (3.14)

From eq. (2.13) of [1],

$$\underline{M}^{1\beta}_{mn} = \underline{u}_\phi e^\beta_{zmn}(y^{1+}, z^+) + \underline{u}_z \frac{\sin\phi_o}{\phi_o} e^\beta_{ymn}(y^{1+}, z^+) \tag{3.15}$$

$$\underline{M}^{2\beta}_{mn} = \underline{u}_\phi e^\beta_{zmn}(y^{2+}, z^+) - \underline{u}_z \frac{\sin\phi_o}{\phi_o} e^\beta_{ymn}(y^{2+}, z^+) \tag{3.16}$$

where e^β_{ymn} and e^β_{zmn} are, respectively, the y- and z-components of $\underline{e}^\beta_{mn}$. Here, β is either TM or TE, and $\underline{e}^{\rm TM}_{mn}$ and $\underline{e}^{\rm TE}_{mn}$ are given by eqs. (A.10) and (A.23) of [1]. In (3.15) and (3.16),

$$y^{1+} = (\pi - \phi)x_o + \tfrac{b}{2} \tag{3.17}$$

$$y^{2+} = \phi x_o + \tfrac{b}{2} \tag{3.18}$$

$$z^+ = z + \tfrac{c}{2} \tag{3.19}$$

where

$$x_o = \frac{a \sin\phi_o}{\phi_o}. \tag{3.20}$$

With $\underline{e}^{\rm TM}_{mn}$ and $\underline{e}^{\rm TE}_{mn}$ given by eqs. (A.10) and (A.23) of [1], (3.15) and (3.16) expand to

$$\begin{aligned}\underline{M}^{1\rm TM}_{mn} = -\frac{2\pi}{k_{mn}\sqrt{bc}}\Bigg\{ & \underline{u}_\phi \left(\frac{n}{c}\right) \sin\left(\frac{m\pi y^{1+}}{b}\right) \cos\left(\frac{n\pi z^+}{c}\right) \\ & + \underline{u}_z \left(\frac{\sin\phi_o}{\phi_o}\right)\left(\frac{m}{b}\right) \cos\left(\frac{m\pi y^{1+}}{b}\right) \sin\left(\frac{n\pi z^+}{c}\right) \Bigg\}\end{aligned} \tag{3.21}$$

$$\underline{M}_{mn}^{1\mathrm{TE}} = \frac{\pi}{k_{mn}}\sqrt{\frac{\epsilon_m\epsilon_n}{bc}}\left\{ -\underline{u}_\phi\left(\frac{m}{b}\right)\sin\left(\frac{m\pi y^{1+}}{b}\right)\cos\left(\frac{n\pi z^+}{c}\right)\right.$$

$$\left. +\underline{u}_z\left(\frac{\sin\phi_o}{\phi_o}\right)\left(\frac{n}{c}\right)\cos\left(\frac{m\pi y^{1+}}{b}\right)\sin\left(\frac{n\pi z^+}{c}\right)\right\} \tag{3.22}$$

$$\underline{M}_{mn}^{2\mathrm{TM}} = -\frac{2\pi}{k_{mn}\sqrt{bc}}\left\{\underline{u}_\phi\left(\frac{n}{c}\right)\sin\left(\frac{m\pi y^{2+}}{b}\right)\cos\left(\frac{n\pi z^+}{c}\right)\right.$$

$$\left. -\underline{u}_z\left(\frac{\sin\phi_o}{\phi_o}\right)\left(\frac{m}{b}\right)\cos\left(\frac{m\pi y^{2+}}{b}\right)\sin\left(\frac{n\pi z^+}{c}\right)\right\} \tag{3.23}$$

$$\underline{M}_{mn}^{2\mathrm{TE}} = \frac{\pi}{k_{mn}}\sqrt{\frac{\epsilon_m\epsilon_n}{bc}}\left\{ -\underline{u}_\phi\left(\frac{m}{b}\right)\sin\left(\frac{m\pi y^{2+}}{b}\right)\cos\left(\frac{n\pi z^+}{c}\right)\right.$$

$$\left. -\underline{u}_z\left(\frac{\sin\phi_o}{\phi_o}\right)\left(\frac{n}{c}\right)\cos\left(\frac{m\pi y^{2+}}{b}\right)\sin\left(\frac{n\pi z^+}{c}\right)\right\}. \tag{3.24}$$

3.4.2 The $\underline{H}$'s in (3.14)

Substitution of eqs. (B.23) and (B.34) of [1] into (3.11) gives

$$\underline{H}_{rs}^{\mathrm{TM}e} = -\sqrt{\frac{\epsilon_r k}{\pi\eta\beta_{rs}^{\mathrm{TM}}}}\left(\frac{2}{aJ_{r+1}(x_{rs})}\right)\left\{\underline{u}_\rho\frac{rJ_r(k_{rs}^{\mathrm{TM}}\rho)\sin(r\phi)}{k_{rs}^{\mathrm{TM}}\rho}\right.$$

$$\left. +\underline{u}_\phi J_r'(k_{rs}^{\mathrm{TM}}\rho)\cos(r\phi)\right\}\cos(\beta_{rs}^{\mathrm{TM}}(L_3 - z)) \tag{3.25}$$

$$\underline{H}_{rs}^{\mathrm{TM}o} = \sqrt{\frac{2k}{\pi\eta\beta_{rs}^{\mathrm{TM}}}}\left(\frac{2}{aJ_{r+1}(x_{rs})}\right)\left\{\underline{u}_\rho\frac{rJ_r(k_{rs}^{\mathrm{TM}}\rho)\cos(r\phi)}{k_{rs}^{\mathrm{TM}}\rho}\right.$$

$$\left. -\underline{u}_\phi J_r'(k_{rs}^{\mathrm{TM}}\rho)\sin(r\phi)\right\}\cos(\beta_{rs}^{\mathrm{TM}}(L_3 - z)). \tag{3.26}$$

In obtaining (3.25) and (3.26), we used

$$Z_{rs}^{\mathrm{TM}eo} = \frac{\eta\beta_{rs}^{\mathrm{TM}}}{k} \tag{3.27}$$

where $k = \omega\sqrt{\mu\epsilon}$ is the wavenumber and $\eta = \sqrt{\mu/\epsilon}$ is the intrinsic impedance. Equation (3.27) results when $\gamma_{rs}^{\mathrm{TM}}$ of (3.3) is substituted into eq. (B.25) of

[1]. Substitution of eqs. (B.41), (B.52), (B.59), and (B.63) of [1] into (3.12) gives

$$\underline{H}_{rs}^{\mathrm{TEe}} = \sqrt{\frac{\epsilon_r \beta_{rs}^{\mathrm{TE}}}{\pi k\eta (x_{rs}'^2 - r^2)}} \left(\frac{2k_{rs}^{\mathrm{TE}}}{J_r(x_{rs}')}\right) \Big\{ [-\underline{u}_\rho J_r'(k_{rs}^{\mathrm{TE}}\rho)\cos(r\phi)$$

$$+\underline{u}_\phi \frac{rJ_r(k_{rs}^{\mathrm{TE}}\rho)}{k_{rs}^{\mathrm{TE}}\rho}\sin(r\phi)]\cos(\beta_{rs}^{\mathrm{TE}}(L_3 - z))$$

$$+\underline{u}_z \left(\frac{k_{rs}^{\mathrm{TE}}}{\beta_{rs}^{\mathrm{TE}}}\right) J_r(k_{rs}^{\mathrm{TE}}\rho)\cos(r\phi)\sin(\beta_{rs}^{\mathrm{TE}}(L_3 - z))\Big\} \tag{3.28}$$

$$\underline{H}_{rs}^{\mathrm{TEo}} = \sqrt{\frac{2\beta_{rs}^{\mathrm{TE}}}{\pi k\eta (x_{rs}'^2 - r^2)}} \left(\frac{2k_{rs}^{\mathrm{TE}}}{J_r(x_{rs}')}\right) \Big\{ [-\underline{u}_\rho J_r'(k_{rs}^{\mathrm{TE}}\rho)\sin(r\phi)$$

$$-\underline{u}_\phi \frac{rJ_r(k_{rs}^{\mathrm{TE}}\rho)}{k_{rs}^{\mathrm{TE}}\rho}\cos(r\phi)]\cos(\beta_{rs}^{\mathrm{TE}}(L_3 - z))$$

$$+\underline{u}_z \left(\frac{k_{rs}^{\mathrm{TE}}}{\beta_{rs}^{\mathrm{TE}}}\right) J_r(k_{rs}^{\mathrm{TE}}\rho)\sin(r\phi)\sin(\beta_{rs}^{\mathrm{TE}}(L_3 - z))\Big\}. \tag{3.29}$$

In obtaining (3.28) and (3.29), we used

$$Y_{rs}^{\mathrm{TEeo}} = \frac{\beta_{rs}^{\mathrm{TE}}}{k\eta}. \tag{3.30}$$

Equation (3.30) results when $\gamma_{rs}^{\mathrm{TE}}$ of (3.4) is substituted into eq. (B.54) of [1].

Because

$$J_r(x_{rs}) = 0, \tag{3.31}$$

substitution of $x = x_{rs}$ into the identity (see formula 9.1.27 of [4])

$$J_{r+1}(x) = -J_r'(x) + \frac{r}{x}J_r(x) \tag{3.32}$$

gives

$$J_{r+1}(x_{rs}) = -J_r'(x_{rs}). \tag{3.33}$$

When $\rho = a$, it follows from (3.5) that

$$k_{rs}^{\mathrm{TM}} \rho = x_{rs} \tag{3.34}$$

and consequently,

$$J_r(k_{rs}^{\mathrm{TM}} \rho) = 0 \tag{3.35}$$

$$J_r'(k_{rs}^{\mathrm{TM}} \rho) = J_r'(x_{rs}). \tag{3.36}$$

Setting $\rho = a$ in (3.25) and (3.26) and using (3.33), (3.35), and (3.36), we obtain

$$\left[\underline{H}_{rs}^{\mathrm{TMe}}\right]_{\rho=a} = \sqrt{\frac{\epsilon_r k}{\pi \eta \beta_{rs}^{\mathrm{TM}}}} \left(\frac{2}{a}\right) \underline{u}_\phi \cos(r\phi) \cos(\beta_{rs}^{\mathrm{TM}}(L_3 - z)) \tag{3.37}$$

$$\left[\underline{H}_{rs}^{\mathrm{TMo}}\right]_{\rho=a} = \sqrt{\frac{2k}{\pi \eta \beta_{rs}^{\mathrm{TM}}}} \left(\frac{2}{a}\right) \underline{u}_\phi \sin(r\phi) \cos(\beta_{rs}^{\mathrm{TM}}(L_3 - z)). \tag{3.38}$$

Recall that

$$J_r'(x_{rs}') = 0. \tag{3.39}$$

When $\rho = a$, it follows from (3.6) that

$$k_{rs}^{\mathrm{TE}} \rho = x_{rs}' \tag{3.40}$$

and consequently,

$$J_r(k_{rs}^{\mathrm{TE}} \rho) = J_r(x_{rs}') \tag{3.41}$$

$$J_r'(k_{rs}^{\mathrm{TE}} \rho) = 0. \tag{3.42}$$

Setting $\rho = a$ in (3.28) and (3.29) and using (3.6) and (3.40)–(3.42), we obtain

$$\left[\underline{H}_{rs}^{\mathrm{TEe}}\right]_{\rho=a} = \sqrt{\frac{\epsilon_r \beta_{rs}^{\mathrm{TE}}}{\pi k \eta (x_{rs}'^2 - r^2)}} \left(\frac{2x_{rs}'}{a}\right) \Big\{ \underline{u}_\phi \left(\frac{r}{x_{rs}'}\right) \sin(r\phi) \cos(\beta_{rs}^{\mathrm{TE}}(L_3 - z))$$

$$+ \underline{u}_z \left(\frac{x_{rs}'}{\beta_{rs}^{\mathrm{TE}} a}\right) \cos(r\phi) \sin(\beta_{rs}^{\mathrm{TE}}(L_3 - z)) \Big\} \tag{3.43}$$

$$\left[H_{rs}^{\mathrm{TE}o}\right]_{\rho=a}=\sqrt{\frac{2\beta_{rs}^{\mathrm{TE}}}{\pi k\eta(x_{rs}'^2-r^2)}}\left(\frac{2x_{rs}'}{a}\right)\left\{-u_\phi\left(\frac{r}{x_{rs}'}\right)\cos(r\phi)\cos(\beta_{rs}^{\mathrm{TE}}(L_3-z))\right.$$

$$\left.+u_z\left(\frac{x_{rs}'}{\beta_{rs}^{\mathrm{TE}}a}\right)\sin(r\phi)\sin(\beta_{rs}^{\mathrm{TE}}(L_3-z))\right\}. \tag{3.44}$$

Wanting to express (3.37), (3.38), (3.43), and (3.44) in terms of y^{1+} and z^+ rather than ϕ and z, we first solve (3.17) for ϕ in terms of y^{1+}:

$$\phi=\pi+\frac{1}{x_o}\left(\frac{b}{2}-y^{1+}\right). \tag{3.45}$$

Defining

$$L_3^+=L_3+\frac{c}{2}, \tag{3.46}$$

and recalling (3.19), we have

$$L_3-z=L_3^+-z^+. \tag{3.47}$$

Substitution of (3.45) and (3.47) into (3.37), (3.38), (3.43), and (3.44) gives

$$\left[H_{rs}^{\mathrm{TM}e}\right]_{\rho=a}=\sqrt{\frac{\epsilon_r k}{\pi\eta\beta_{rs}^{\mathrm{TM}}}}\left(\frac{2}{a}\right)(-1)^r u_\phi\cos\left(\frac{r}{x_o}(\tfrac{b}{2}-y^{1+})\right)\cos(\beta_{rs}^{\mathrm{TM}}(L_3^+-z^+)) \tag{3.48}$$

$$\left[H_{rs}^{\mathrm{TM}o}\right]_{\rho=a}=\sqrt{\frac{2k}{\pi\eta\beta_{rs}^{\mathrm{TM}}}}\left(\frac{2}{a}\right)(-1)^r u_\phi\sin\left(\frac{r}{x_o}(\tfrac{b}{2}-y^{1+})\right)\cos(\beta_{rs}^{\mathrm{TM}}(L_3^+-z^+)) \tag{3.49}$$

$$\left[H_{rs}^{\mathrm{TE}e}\right]_{\rho=a}=\sqrt{\frac{\epsilon_r\beta_{rs}^{\mathrm{TE}}}{\pi k\eta\,(x_{rs}'^2-r^2)}}\left(\frac{2x_{rs}'}{a}\right)(-1)^r\left\{u_\phi\left(\frac{r}{x_{rs}'}\right)\sin\left(\frac{r}{x_o}(\tfrac{b}{2}-y^{1+})\right)\right.$$

$$\left.\cdot\cos(\beta_{rs}^{\mathrm{TE}}(L_3^+-z^+))+u_z\left(\frac{x_{rs}'}{\beta_{rs}^{\mathrm{TE}}a}\right)\cos\left(\frac{r}{x_o}(\tfrac{b}{2}-y^{1+})\right)\sin(\beta_{rs}^{\mathrm{TE}}(L_3^+-z^+))\right\} \tag{3.50}$$

$$\left[H_{rs}^{\mathrm{TE}o}\right]_{\rho=a}=\sqrt{\frac{2\beta_{rs}^{\mathrm{TE}}}{\pi k\eta\,(x_{rs}'^2-r^2)}}\left(\frac{2x_{rs}'}{a}\right)(-1)^r\left\{-u_\phi\left(\frac{r}{x_{rs}'}\right)\cos\left(\frac{r}{x_o}(\tfrac{b}{2}-y^{1+})\right)\right.$$

$$\cdot \cos(\beta_{rs}^{\mathrm{TE}}(L_3^+ - z^+)) + \underline{u}_z \left(\frac{x'_{rs}}{\beta_{rs}^{\mathrm{TE}} a}\right) \sin\left(\frac{r}{x_o}\left(\tfrac{b}{2} - y^{1+}\right)\right) \sin(\beta_{rs}^{\mathrm{TE}}(L_3^+ - z^+))\Big\}. \tag{3.51}$$

Expressions (3.48)–(3.51) are valid only in A_1. To obtain expressions valid in A_2, we first solve (3.18) for ϕ in terms of y^{2+}:

$$\phi = \frac{1}{x_o}\left(y^{2+} - \tfrac{b}{2}\right). \tag{3.52}$$

Substitution of (3.52) and (3.47) into (3.37), (3.38), (3.43), and (3.44) gives

$$\left[\underline{H}_{rs}^{\mathrm{TMe}}\right]_{\rho=a} = \sqrt{\frac{\epsilon_r k}{\pi\eta\beta_{rs}^{\mathrm{TM}}}}\left(\frac{2}{a}\right)\underline{u}_\phi \cos\left(\frac{r}{x_o}\left(y^{2+} - \tfrac{b}{2}\right)\right)\cos(\beta_{rs}^{\mathrm{TM}}(L_3^+ - z^+)) \tag{3.53}$$

$$\left[\underline{H}_{rs}^{\mathrm{TMo}}\right]_{\rho=a} = \sqrt{\frac{2k}{\pi\eta\beta_{rs}^{\mathrm{TM}}}}\left(\frac{2}{a}\right)\underline{u}_\phi \sin\left(\frac{r}{x_o}\left(y^{2+} - \tfrac{b}{2}\right)\right)\cos(\beta_{rs}^{\mathrm{TM}}(L_3^+ - z^+)) \tag{3.54}$$

$$\left[\underline{H}_{rs}^{\mathrm{TEe}}\right]_{\rho=a} = \sqrt{\frac{\epsilon_r\beta_{rs}^{\mathrm{TE}}}{\pi k\eta\,(x'^2_{rs} - r^2)}}\left(\frac{2x'_{rs}}{a}\right)\Big\{\underline{u}_\phi\left(\frac{r}{x'_{rs}}\right)\sin\left(\frac{r}{x_o}\left(y^{2+} - \tfrac{b}{2}\right)\right)$$

$$\cdot \cos(\beta_{rs}^{\mathrm{TE}}(L_3^+ - z^+)) + \underline{u}_z\left(\frac{x'_{rs}}{\beta_{rs}^{\mathrm{TE}} a}\right)\cos\left(\frac{r}{x_o}\left(y^{2+} - \tfrac{b}{2}\right)\right)\sin(\beta_{rs}^{\mathrm{TE}}(L_3^+ - z^+))\Big\} \tag{3.55}$$

$$\left[\underline{H}_{rs}^{\mathrm{TEo}}\right]_{\rho=a} = \sqrt{\frac{2\beta_{rs}^{\mathrm{TE}}}{\pi k\eta\,(x'^2_{rs} - r^2)}}\left(\frac{2x'_{rs}}{a}\right)\Big\{-\underline{u}_\phi\left(\frac{r}{x'_{rs}}\right)\cos\left(\frac{r}{x_o}\left(y^{2+} - \tfrac{b}{2}\right)\right)$$

$$\cdot \cos(\beta_{rs}^{\mathrm{TE}}(L_3^+ - z^+)) + \underline{u}_z\left(\frac{x'_{rs}}{\beta_{rs}^{\mathrm{TE}} a}\right)\sin\left(\frac{r}{x_o}\left(y^{2+} - \tfrac{b}{2}\right)\right)\sin(\beta_{rs}^{\mathrm{TE}}(L_3^+ - z^+))\Big\}. \tag{3.56}$$

Expressions (3.53)–(3.56) are valid in A_2.

3.4.3 Substitution of the $\underline{M}$'s and $\underline{H}$'s of Sections 3.4.1 and 3.4.2 into (3.14)

In (3.14), $ds = a\,d\phi\,dz$. Substituting (3.21), (3.22), and (3.48) into (3.14) and changing the variables of integration from ϕ and z to y^{1+} and z^+, we

obtain

$$I_i^{1\mathrm{TM},\mathrm{TM}e} = (-1)^r n I_{i,rs}^{\mathrm{TM}e} \tag{3.57}$$

$$I_i^{1\mathrm{TE},\mathrm{TM}e} = (-1)^r \left(\frac{mc}{b}\right) I_{i,rs}^{\mathrm{TM}e} \tag{3.58}$$

where

$$I_{i,rs}^{\mathrm{TM}e} = T^{\mathrm{TM}} y_{sc} z_{cc}^{\mathrm{TM}} \tag{3.59}$$

in which

$$T^{\mathrm{TM}} = \left(\frac{2}{k_{mn}b}\right)\sqrt{\left(\frac{\epsilon_m\epsilon_n}{4}\right)\left(\frac{b}{c}\right)}\sqrt{(2\pi)\left(\frac{\epsilon_r}{2}\right)\left(\frac{k}{\beta_{rs}^{\mathrm{TM}}}\right)} \tag{3.60}$$

$$y_{sc} = \frac{2\phi_o}{b}\int_0^b \sin\left(\frac{m\pi y^{1+}}{b}\right)\cos\left(\frac{r}{x_o}(y^{1+}-\tfrac{b}{2})\right)dy^{1+} \tag{3.61}$$

$$z_{cc}^{\mathrm{TM}} = \frac{2}{c}\int_0^c \cos\left(\frac{n\pi z^+}{c}\right)\cos\left(\beta_{rs}^{\mathrm{TM}}(L_3^+ - z^+)\right)dz^+. \tag{3.62}$$

In obtaining (3.57), we replaced 2 by $\sqrt{\epsilon_m\epsilon_n}$ in (3.21). Such replacement is allowed because the right-hand side of (3.21) vanishes when either n or m is zero. The coefficient $I_{i,rs}^{\mathrm{TM}e}$ is so named because it is proportional to the quantity that the $\underline{e}$ven $\underline{\mathrm{TM}}_{\underline{rs}}$ modal part of the incident magnetic field in the circular waveguide contributes to $-j\sqrt{\eta}I_i^{1\mathrm{TM}}$ and $-j\sqrt{\eta}I_i^{1\mathrm{TE}}$ of (3.13).[†] As defined by (3.61), y_{sc} is an integral with respect to $\underline{y}^{1+}$ of the product of a $\underline{s}$ine and a $\underline{c}$osine. As defined by (3.62), z_{cc}^{TM} is an integral with respect to $\underline{z}^+$ of a $\underline{c}$osine and a $\underline{c}$osine whose argument is proportional to the $\underline{\mathrm{TM}}$ propagation constant β_{rs}^{TM}. Substitution of (3.23), (3.24), and (3.53) into (3.14) gives

$$I_i^{2\mathrm{TM},\mathrm{TM}e} = n I_{i,rs}^{\mathrm{TM}e} \tag{3.63}$$

$$I_i^{2\mathrm{TE},\mathrm{TM}e} = \left(\frac{mc}{b}\right) I_{i,rs}^{\mathrm{TM}e} \tag{3.64}$$

[†]Rather than indicating vectors, the underlines in this sentence highlight the superscripts and subscripts of $I_{i,rs}^{\mathrm{TM}e}$.

where $I_{i,rs}^{\mathrm{TMe}}$ is given by (3.59) in which y_{ec} is given by the right-hand side of (3.61) with y^{1+} replaced by y^{2+}. Since y^{1+} and y^{2+} are dummy variables of integration, it follows that the y_{ec} implicit in (3.63) and (3.64) is given by the unaltered right-hand side of (3.61). Therefore, the $I_{i,rs}^{\mathrm{TMe}}$ in (3.63) and (3.64) is precisely that given by (3.59) in which T^{TM}, y_{ec}, and z_{ec}^{TM} are given by (3.60), (3.61), and (3.62), respectively. In obtaining (3.63), we replaced 2 by $\sqrt{\epsilon_m \epsilon_n}$ in (3.23). Such replacement is allowed because the right-hand side of (3.23) vanishes when either n or m is zero.

Substituting (3.21), (3.22), and (3.49) into (3.14), we obtain

$$I_i^{\mathrm{1TM,TMo}} = (-1)^{r+1} n I_{i,rs}^{\mathrm{TMo}} \tag{3.65}$$

$$I_i^{\mathrm{1TE,TMo}} = (-1)^{r+1} \left(\frac{mc}{b}\right) I_{i,rs}^{\mathrm{TMo}} \tag{3.66}$$

where

$$I_{i,rs}^{\mathrm{TMo}} = T^{\mathrm{TM}} y_{os} z_{ec}^{\mathrm{TM}} \tag{3.67}$$

$$y_{os} = \frac{2\phi_o}{b} \int_0^b \sin\left(\frac{m\pi y^{1+}}{b}\right) \sin\left(\frac{r}{x_o}(y^{1+} - \tfrac{b}{2})\right) dy^{1+}. \tag{3.68}$$

In (3.67), T^{TM} and z_{ec}^{TM} are given by (3.60) and (3.62). In obtaining (3.65) and (3.66), we replaced 2 by ϵ_r in (3.49). Such replacement is allowed because the right-hand side of (3.49) vanishes when $r = 0$. The coefficient $I_{i,rs}^{\mathrm{TMo}}$ is so named because it is proportional to the quantity that the odd TM_{rs} modal part of the incident magnetic field in the circular waveguide contributes to $-j\sqrt{\eta} I_i^{\mathrm{1TM}}$ and $-j\sqrt{\eta} I_i^{\mathrm{1TE}}$ of (3.13). As defined by (3.68), y_{os} is an integral with respect to y^{1+} of the product of a sine and a sine. Substitution of (3.23), (3.24), and (3.54) into (3.14) gives

$$I_i^{\mathrm{2TM,TMo}} = n I_{i,rs}^{\mathrm{TMo}} \tag{3.69}$$

$$I_i^{\mathrm{2TE,TMo}} = \left(\frac{mc}{b}\right) I_{i,rs}^{\mathrm{TMo}}. \tag{3.70}$$

In obtaining (3.69) and (3.70), we replaced 2 by ϵ_r in (3.54). Such replacement is allowed because the right-hand side of (3.54) vanishes when $r = 0$.

Substituting (3.21), (3.22), and (3.50) into (3.14), we obtain

$$I_i^{\mathrm{1TM,TEo}} = (-1)^{r+1} \left\{ n I_{i,rs}^{\mathrm{TEo\phi}} + \left(\frac{mc}{b}\right) I_{i,rs}^{\mathrm{TEez}} \right\} \tag{3.71}$$

$$I_i^{\mathrm{1TE,TEo}} = (-1)^{r+1} \left\{ \left(\frac{mc}{b}\right) I_{i,rs}^{\mathrm{TEo\phi}} - n I_{i,rs}^{\mathrm{TEez}} \right\} \tag{3.72}$$

where

$$I_{i,rs}^{\mathrm{TEe}\phi} = T^{\mathrm{TE}} r y_{ss} z_{cc}^{\mathrm{TE}} \tag{3.73}$$

$$I_{i,rs}^{\mathrm{TEe}z} = -T^{\mathrm{TE}} \left(\frac{\sin\phi_o}{\phi_o}\right) \left(\frac{x_{rs}'^2}{\beta_{rs}^{\mathrm{TE}} a}\right) y_{cc} z_{ss}^{\mathrm{TE}} \tag{3.74}$$

$$T^{\mathrm{TE}} = \left(\frac{2}{k_{mn} b}\right) \sqrt{\left(\frac{\epsilon_m \epsilon_n}{4}\right)\left(\frac{b}{c}\right)} \sqrt{(2\pi)\left(\frac{\epsilon_r}{2}\right)\left(\frac{\beta_{rs}^{\mathrm{TE}}}{k}\right)\left(\frac{1}{x_{rs}'^2 - r^2}\right)} \tag{3.75}$$

$$y_{cc} = \frac{2\phi_o}{b} \int_0^b \cos\left(\frac{m\pi y^{1+}}{b}\right) \cos\left(\frac{r}{x_o}(y^{1+} - \tfrac{b}{2})\right) dy^{1+} \tag{3.76}$$

$$z_{cc}^{\mathrm{TE}} = \frac{2}{c} \int_0^c \cos\left(\frac{n\pi z^+}{c}\right) \cos\left(\beta_{rs}^{\mathrm{TE}}(L_3^+ - z^+)\right) dz^+ \tag{3.77}$$

$$z_{ss}^{\mathrm{TE}} = \frac{2}{c} \int_0^c \sin\left(\frac{n\pi z^+}{c}\right) \sin\left(\beta_{rs}^{\mathrm{TE}}(L_3^+ - z^+)\right) dz^+ . \tag{3.78}$$

In (3.73), y_{ss} is given by (3.68). The quantity $I_{i,rs}^{\mathrm{TEe}\phi}$ is proportional to the quantity that the $\underline{\phi}$-component of the $\underline{e}$ven $\underline{\mathrm{TE}}_{rs}$ modal part of the incident magnetic field contributes to $-j\sqrt{\eta}I_i^{1\mathrm{TM}}$ of (3.13). The quantity $I_{i,rs}^{\mathrm{TEe}z}$ is proportional to the quantity that the $\underline{z}$-component of the $\underline{e}$ven $\underline{\mathrm{TE}}_{rs}$ modal part of the incident magnetic field contributes to $-j\sqrt{\eta}I_i^{1\mathrm{TM}}$ of (3.13). Substituting (3.23), (3.24), and (3.55) into (3.14), we obtain

$$I_i^{2\mathrm{TM},\mathrm{TEe}} = (-1)^{r+1} I_i^{1\mathrm{TM},\mathrm{TEe}} \tag{3.79}$$

$$I_i^{2\mathrm{TE},\mathrm{TEe}} = (-1)^{r+1} I_i^{1\mathrm{TE},\mathrm{TEe}} . \tag{3.80}$$

Substitution of (3.21), (3.22), and (3.51) into (3.14) gives

$$I_i^{1\mathrm{TM},\mathrm{TEo}} = (-1)^{r+1} \left\{ n I_{i,rs}^{\mathrm{TEo}\phi} - \left(\frac{mc}{b}\right) I_{i,rs}^{\mathrm{TEo}z} \right\} \tag{3.81}$$

$$I_i^{1\mathrm{TE},\mathrm{TEo}} = (-1)^{r+1} \left\{ \left(\frac{mc}{b}\right) I_{i,rs}^{\mathrm{TEo}\phi} + n I_{i,rs}^{\mathrm{TEo}z} \right\} \tag{3.82}$$

where

$$I_{i,rs}^{\mathrm{TEo}\phi} = T^{\mathrm{TE}} r y_{sc} z_{cc}^{\mathrm{TE}} \tag{3.83}$$

$$I_{i,rs}^{\mathrm{TEo}z} = -T^{\mathrm{TE}} \left(\frac{\sin\phi_o}{\phi_o}\right) \left(\frac{x_{rs}'^2}{\beta_{rs}^{\mathrm{TE}} a}\right) y_{cs} z_{ss}^{\mathrm{TE}} \tag{3.84}$$

where

$$y_{cs} = \frac{2\phi_o}{b} \int_0^b \cos\left(\frac{m\pi y^{1+}}{b}\right) \sin\left(\frac{r}{x_o}(y^{1+} - \tfrac{b}{2})\right) dy^{1+}. \tag{3.85}$$

In (3.83) and (3.84), T^{TE}, y_{sc}, z^{TE}_{cc}, and z^{TE}_{ss} are given by (3.75), (3.61), (3.77), and (3.78), respectively. In obtaining (3.81) and (3.82), we replaced 2 by ϵ_r in (3.51). Such replacement is allowed because the right-hand side of (3.51) vanishes when $r = 0$. The quantity $I^{TEo\phi}_{i,rs}$ is proportional to the quantity that the ϕ-component of the odd $\underline{\mathrm{TE}}_{rs}$ modal part of the incident magnetic field contributes to $-j\sqrt{\eta}I^{1TM}_{i}$ of (3.13). The quantity $I^{TEoz}_{i,rs}$ is proportional to the quantity that the z-component of the odd $\underline{\mathrm{TE}}_{rs}$ modal part of the incident magnetic field contributes to $-j\sqrt{\eta}I^{1TM}_{i}$ of (3.13). Substituting (3.23), (3.24), and (3.56) into (3.14), we obtain

$$I^{2TM,TEo}_{i} = (-1)^r I^{1TM,TEo}_{i} \tag{3.86}$$

$$I^{2TE,TEo}_{i} = (-1)^r I^{1TE,TEo}_{i}. \tag{3.87}$$

In obtaining (3.86) and (3.87), we replaced 2 by ϵ_r in (3.56). Such replacement is allowed because the right-hand side of (3.56) vanishes when $r = 0$.

3.4.4 Evaluation of the y's Defined by (3.61), (3.68), (3.76), and (3.85) of Section 3.4.3

Since

$$\cos\left(\frac{r}{x_o}(y^{1+} - \tfrac{b}{2})\right) = \cos(r\phi_o)\cos\left(\frac{ry^{1+}}{x_o}\right) + \sin(r\phi_o)\sin\left(\frac{ry^{1+}}{x_o}\right) \tag{3.88}$$

where, in agreement with eq. (E.9) of [1],

$$\phi_o = \frac{b}{2x_o}, \tag{3.89}$$

(3.61) and (3.76) become

$$y_{sc} = \phi^{(2)}_m \cos(r\phi_o) + \phi^{(1)}_m \sin(r\phi_o) \tag{3.90}$$

$$y_{cc} = \phi^{(4)}_m \cos(r\phi_o) + \phi^{(3)}_m \sin(r\phi_o) \tag{3.91}$$

where $\phi_m^{(1)}$, $\phi_m^{(2)}$, $\phi_m^{(3)}$, and $\phi_m^{(4)}$ are, respectively, $\phi_p^{(1)}$, $\phi_p^{(2)}$, $\phi_p^{(3)}$, and $\phi_p^{(4)}$ of eqs. (E.14)–(E.17) of [1] with p replaced by m. Since

$$\sin\left(\frac{r}{x_o}(y^{1+} - \tfrac{b}{2})\right) = \cos(r\phi_o)\sin\left(\frac{ry^{1+}}{x_o}\right) - \sin(r\phi_o)\cos\left(\frac{ry^{1+}}{x_o}\right) \quad (3.92)$$

where ϕ_o is given by (3.89), (3.68) and (3.85) become

$$y_{ss} = \phi_m^{(1)}\cos(r\phi_o) - \phi_m^{(2)}\sin(r\phi_o) \quad (3.93)$$

$$y_{cs} = \phi_m^{(3)}\cos(r\phi_o) - \phi_m^{(4)}\sin(r\phi_o)\,. \quad (3.94)$$

Equations (3.90), (3.91), (3.93), and (3.94) can be rewritten as

$$y_{sc} = \phi_m^{b2} \quad (3.95)$$

$$y_{cc} = \phi_m^{b4} \quad (3.96)$$

$$y_{ss} = \phi_m^{b1} \quad (3.97)$$

$$y_{cs} = \phi_m^{b3} \quad (3.98)$$

where ϕ_m^{b1}, ϕ_m^{b2}, ϕ_m^{b3}, and ϕ_m^{b4} are given, respectively, by eqs. (6.28)–(6.31) of [2] with p replaced by m.

3.4.5 Evaluation of the z's Defined by (3.62), (3.77), and (3.78) of Section 3.4.3

Equations (3.62) and (3.77) are written compactly as

$$z_{cc}^{\delta} = \frac{2}{c}\int_0^c \cos\left(\frac{n\pi z^+}{c}\right)\cos(\beta_{rs}^{\delta}(L_3^+ - z^+))\,dz^+, \quad \delta = \mathrm{TM},\mathrm{TE} \quad (3.99)$$

which expands to (see formula 401.06 of [5])

$$z_{cc}^{\delta} = \frac{1}{c}\int_0^c \left\{\cos(n^{\delta+}z^+ - \beta_{rs}^{\delta}L_3^+) + \cos(n^{\delta-}z^+ + \beta_{rs}^{\delta}L_3^+)\right\}dz^+ \quad (3.100)$$

where

$$n^{\delta+} = \frac{n\pi}{c} + \beta_{rs}^{\delta} \quad (3.101)$$

$$n^{\delta-} = \frac{n\pi}{c} - \beta_{rs}^{\delta}\,. \quad (3.102)$$

Performing the integration in (3.100), we obtain

$$z_{cc}^{\delta} = \frac{\sin(n^{\delta+}c - \beta_{rs}^{\delta} L_3^+) + \sin(\beta_{rs}^{\delta} L_3^+)}{n^{\delta+}c} + \frac{\sin(n^{\delta-}c + \beta_{rs}^{\delta} L_3^+) - \sin(\beta_{rs}^{\delta} L_3^+)}{n^{\delta-}c}. \tag{3.103}$$

Concerned about roundoff error when $|n^{\delta\pm}|$ is small, we use formulas 401.01 and 401.02 of [5] and the identity

$$\cos x = 1 - 2\sin^2\left(\frac{x}{2}\right) \tag{3.104}$$

to recast (3.103) as

$$\begin{aligned} z_{cc}^{\delta} &= \frac{\sin(n^{\delta+}c)\cos(\beta_{rs}^{\delta} L_3^+) + 2\sin^2\left(\frac{n^{\delta+}c}{2}\right)\sin(\beta_{rs}^{\delta} L_3^+)}{n^{\delta+}c} \\ &+ \frac{\sin(n^{\delta-}c)\cos(\beta_{rs}^{\delta} L_3^+) - 2\sin^2\left(\frac{n^{\delta-}c}{2}\right)\sin(\beta_{rs}^{\delta} L_3^+)}{n^{\delta-}c} \end{aligned} \tag{3.105}$$

which can be rewritten as

$$z_{cc}^{\delta} = \hat{G}_n^{\delta} \tag{3.106}$$

where $\hat{G}_n^{\delta}$ is the right-hand side of eq. (3.76) of [2] with q replaced by n.

Applying formula (401.07) of [5] to (3.78), we obtain

$$z_{ss}^{\mathrm{TE}} = \frac{1}{c}\int_0^c \left\{\cos(n^{\mathrm{TE}+}z^+ - \beta_{rs}^{\mathrm{TE}} L_3^+) - \cos(n^{\mathrm{TE}-}z^+ + \beta_{rs}^{\mathrm{TE}} L_3^+)\right\} dz^+ . \tag{3.107}$$

The evaluation of (3.107) is similar to that of (3.100):

$$\begin{aligned} z_{ss}^{\mathrm{TE}} &= \frac{\sin(n^{\mathrm{TE}+}c)\cos(\beta_{rs}^{\mathrm{TE}} L_3^+) + 2\sin^2\left(\frac{n^{\mathrm{TE}+}c}{2}\right)\sin(\beta_{rs}^{\mathrm{TE}} L_3^+)}{n^{\mathrm{TE}+}c} \\ &- \frac{\sin(n^{\mathrm{TE}-}c)\cos(\beta_{rs}^{\mathrm{TE}} L_3^+) - 2\sin^2\left(\frac{n^{\mathrm{TE}-}c}{2}\right)\sin(\beta_{rs}^{\mathrm{TE}} L_3^+)}{n^{\mathrm{TE}-}c}. \end{aligned} \tag{3.108}$$

Equation (3.108) can be rewritten as

$$z_{ss}^{\mathrm{TE}} = -\hat{G}_n^{(4)} \tag{3.109}$$

where $\hat{G}_n^{(4)}$ is the right-hand side of eq. (3.108) of [2] with q replaced by n.

3.4.6 Utilization of the Newly Evaluated y's and z's

Substituting (3.95) and (3.106) into (3.59), we obtain

$$I_{i,rs}^{\mathrm{TM}e} = T^{\mathrm{TM}} \phi_m^{b2} \hat{G}_n^{\mathrm{TM}} \tag{3.110}$$

where T^{TM} is given by (3.60). Substitution of (3.97) and (3.106) into (3.67) gives

$$I_{i,rs}^{\mathrm{TM}o} = T^{\mathrm{TM}} \phi_m^{b1} \hat{G}_n^{\mathrm{TM}} . \tag{3.111}$$

Substitution of (3.97) and (3.106) into (3.73), substitution of (3.96) and (3.109) into (3.74), substitution of (3.95) and (3.106) into (3.83), and substitution of (3.98) and (3.109) into (3.84) yield

$$I_{i,rs}^{\mathrm{TE}e\phi} = T^{\mathrm{TE}} r \phi_m^{b1} \hat{G}_n^{\mathrm{TE}} \tag{3.112}$$

$$I_{i,rs}^{\mathrm{TE}ez} = T^{\mathrm{TE}} \left(\frac{\sin\phi_o}{\phi_o}\right) \left(\frac{x_{rs}'^2}{\beta_{rs}^{\mathrm{TE}} a}\right) \phi_m^{b4} \hat{G}_n^{(4)} \tag{3.113}$$

$$I_{i,rs}^{\mathrm{TE}o\phi} = T^{\mathrm{TE}} r \phi_m^{b2} \hat{G}_n^{\mathrm{TE}} \tag{3.114}$$

$$I_{i,rs}^{\mathrm{TE}oz} = T^{\mathrm{TE}} \left(\frac{\sin\phi_o}{\phi_o}\right) \left(\frac{x_{rs}'^2}{\beta_{rs}^{\mathrm{TE}} a}\right) \phi_m^{b3} \hat{G}_n^{(4)} \tag{3.115}$$

where T^{TE} is given by (3.75). In terms of the I's of (3.110)–(3.115), the final expressions for $I_i^{\alpha\beta,\delta f}$ of (3.14) are (3.57), (3.58), (3.63)–(3.66), (3.69)–(3.72), (3.79)–(3.82), (3.86), and (3.87).

3.5 Comparison of the Elements of the Excitation Vector with Those of [2]

The elements $\{-j\sqrt{\eta}\, I_i^{\alpha\beta}\}$ rather than the elements $\{I_i^{\alpha\beta}\}$ of the excitation vector itself are calculated and stored in the array TI in the computer program of the present report. If $B_{01}^{\mathrm{TM}e} = 1$ and all the other B's in (3.13) are zero, then (3.13) reduces to

$$-j\sqrt{\eta}\, I_i^{\alpha\beta} = -j I_i^{\alpha\beta,\mathrm{TM}e}, \quad \begin{cases} \alpha = 1,2 \\ \beta = \mathrm{TM},\mathrm{TE}. \end{cases} \tag{3.116}$$

On the other hand, the elements $\{-jI_i^{\alpha \mathrm{TM}} e^{j\beta_{01}^{\mathrm{TM}} L_3}\}$ and $\{-jI_i^{\alpha \mathrm{TE}} e^{j\beta_{01}^{\mathrm{TM}} L_3}\}$ given by eqs. (4.8) and (4.9) of [2] are calculated and stored in the array TI in the computer program of [2]. Henceforth, the elements mentioned in the previous sentence will be called $\{-j\hat{I}_i^{\alpha \mathrm{TM}} e^{j\beta_{01}^{\mathrm{TM}} L_3}\}$ and $\{-j\hat{I}_i^{\alpha \mathrm{TE}} e^{j\beta_{01}^{\mathrm{TM}} L_3}\}$ and written concisely as $\{-j\hat{I}_i^{\alpha\beta} e^{j\beta_{01}^{\mathrm{TM}} L_3}\}$ where the β that appears in $\alpha\beta$ is either TM or TE. In this section, we relate the elements $\{-j\sqrt{\eta} I_i^{\alpha\beta}\}$ of (3.116) to the elements $\{-j\hat{I}_i^{\alpha\beta} e^{j\beta_{01}^{\mathrm{TM}} L_3}\}$.

When $B_{01}^{\mathrm{TM}e} = 1$ and all the other B's are zero, the excitation in the present report is, according to (3.2), the z-traveling wave whose magnetic field is $\underline{H}^{(3+)}(\underline{J}^{\mathrm{imp}}, \underline{0})$ given by

$$\underline{H}^{(3+)}(\underline{J}^{\mathrm{imp}}, \underline{0}) = \frac{e^{j\beta_{01}^{\mathrm{TM}} L_3}}{\sqrt{Z_{01}^{\mathrm{TM}eo}}} \underline{H}_{01}^{\mathrm{TM}e+} . \tag{3.117}$$

On the other hand, the excitation in [2] is the z-traveling wave whose magnetic field is $\underline{H}_{01}^{\mathrm{TM}e+}$ given by eq. (5.2) of [1].† Hence, the excitation of the present report is that of [2] multiplied by $e^{j\beta_{01}^{\mathrm{TM}} L_3}/\sqrt{Z_{01}^{\mathrm{TM}eo}}$. Since the elements of the excitation vector are proportional to the excitation, it follows that

$$I_i^{\alpha\beta} = \frac{e^{j\beta_{01}^{\mathrm{TM}} L_3}}{\sqrt{Z_{01}^{\mathrm{TM}eo}}} \hat{I}_i^{\alpha\beta}. \tag{3.118}$$

Multiplying both sides of (3.118) by $-j\sqrt{\eta}$ and using (3.27), we obtain

$$-j\sqrt{\eta} I_i^{\alpha\beta} = \sqrt{\frac{k}{\beta_{01}^{\mathrm{TM}}}} \left(-j\hat{I}_i^{\alpha\beta} e^{j\beta_{01}^{\mathrm{TM}} L_3}\right) . \tag{3.119}$$

In Appendix A, we verify that $-j\sqrt{\eta} I_i^{\alpha\beta}$ as given by (3.116) satisfies (3.119) with $-j\hat{I}_i^{\alpha\beta} e^{j\beta_{01}^{\mathrm{TM}} L_3}$ given by the right-hand sides of eqs. (4.8) and (4.9) of [2].

†The excitation in [2] is the same as that in [1].

Chapter 4

The Propagating Modes in the Rectangular Waveguides

4.1 The Normalized Coefficients of the Propagating Modes in the Rectangular Waveguides

Dividing both sides of eqs. (5.30) and (5.31) of [2] by $e^{j\beta_{01}^{TM}L_3}/\sqrt{Z_{01}^{TMeo}}$, we obtain expressions for the electric fields $\underline{E}^{(1)}$ and $\underline{E}^{(2)}$ in the rectangular waveguides:

$$\underline{E}^{(1)} = \frac{C_{10}^{1TE-} e^{j\beta_{10}z_o} \underline{E}_{10}^{TE-} + C_{10}^{1TE+} e^{-j\beta_{10}z_o} \underline{E}_{10}^{TE+}}{\sqrt{Y_{10}^{TE}}} + \text{ evanescent waves} \quad (4.1)$$

$$\underline{E}^{(2)} = \frac{C_{10}^{2TE+} e^{j\beta_{10}z_o} \underline{E}_{10}^{TE+} + C_{10}^{2TE-} e^{-j\beta_{10}z_o} \underline{E}_{10}^{TE-}}{\sqrt{Y_{10}^{TE}}} + \text{ evanescent waves} \quad (4.2)$$

where the C's are those of [2] divided by $e^{j\beta_{01}^{TM}L_3}/\sqrt{Z_{01}^{TMeo}}$. The evanescent waves in (4.1) are those of all the evanescent modes in the left-hand rectangular waveguide in Fig. 2. The evanescent waves in (4.2) are those of all the evanescent modes in the right-hand rectangular waveguide in Fig. 2. In view of (3.27) and eqs. (5.32), (5.33), (5.36), and (5.37) of [2],† the C's in (4.1)

†The left-hand side of eq. (5.37) of [2] should be C_{10}^{2TE-}.

and (4.2) are given by

$$C_{10}^{1\mathrm{TE}-} = \left(\sqrt{\frac{\beta_{10}}{k}}\right)\left\{\frac{(Z_1 Y_{10}^{\mathrm{TE}}+1)e^{j\beta_{10}z_1}}{2(Z_1 Y_{10}^{\mathrm{TE}}\cos(\beta_{10}z_1)+j\sin(\beta_{10}z_1))}\right\}\left(\frac{V_{10}^{1\mathrm{TE}}}{\sqrt{\eta}}\right) \tag{4.3}$$

$$C_{10}^{1\mathrm{TE}+} = \left(\sqrt{\frac{\beta_{10}}{k}}\right)\left\{\frac{(Z_1 Y_{10}^{\mathrm{TE}}-1)e^{-j\beta_{10}z_1}}{2(Z_1 Y_{10}^{\mathrm{TE}}\cos(\beta_{10}z_1)+j\sin(\beta_{10}z_1))}\right\}\left(\frac{V_{10}^{1\mathrm{TE}}}{\sqrt{\eta}}\right) \tag{4.4}$$

$$C_{10}^{2\mathrm{TE}+} = \left(\sqrt{\frac{\beta_{10}}{k}}\right)\left\{\frac{(Z_2 Y_{10}^{\mathrm{TE}}+1)e^{j\beta_{10}z_2}}{2(Z_2 Y_{10}^{\mathrm{TE}}\cos(\beta_{10}z_2)+j\sin(\beta_{10}z_2))}\right\}\left(\frac{V_{10}^{2\mathrm{TE}}}{\sqrt{\eta}}\right) \tag{4.5}$$

$$C_{10}^{2\mathrm{TE}-} = \left(\sqrt{\frac{\beta_{10}}{k}}\right)\left\{\frac{(Z_2 Y_{10}^{\mathrm{TE}}-1)e^{-j\beta_{10}z_2}}{2(Z_2 Y_{10}^{\mathrm{TE}}\cos(\beta_{10}z_2)+j\sin(\beta_{10}z_2))}\right\}\left(\frac{V_{10}^{2\mathrm{TE}}}{\sqrt{\eta}}\right). \tag{4.6}$$

The C's given by (4.3)–(4.6) are the normalized coefficients of the propagating modes in the rectangular waveguides.

4.2 The Time-Average Power of the Propagating Modes in the Rectangular Waveguides

The time-average power transmitted into the left-hand rectangular waveguide of Fig. 2 is $P_t^{(1)}$ given by

$$P_t^{(1)} = \left|C_{10}^{1\mathrm{TE}-}\right|^2 - \left|C_{10}^{1\mathrm{TE}+}\right|^2. \tag{4.7}$$

Here, the subscript "t" stands for "transmitted". The time-average power (4.7) is also the time-average power radiated by the magnetic current $\underline{M}^{(1)}$ in the left-hand rectangular waveguide of Fig. 2.

The time-average power tansmitted into the right-hand rectangular waveguide of Fig. 2 is $P_t^{(2)}$ given by

$$P_t^{(2)} = \left|C_{10}^{2\mathrm{TE}+}\right|^2 - \left|C_{10}^{2\mathrm{TE}-}\right|^2. \tag{4.8}$$

The time average power (4.8) is also the time-average power radiated by the magnetic current $\underline{M}^{(2)}$ in the right-hand rectangular waveguide of Fig. 2.

4.3 Alternative Expressions for the Time-Average Power Radiated by the Magnetic Currents in the Rectangular Waveguides

An alternative expression for the time-average power radiated by the magnetic current $\underline{M}^{(1)}$ in the left-hand rectangular waveguide of Fig. 2 is $P_{ta}^{(1)}$ given by

$$P_{ta}^{(1)} = -\mathrm{Re}\left\{\iint_{A_1} \left(\underline{M}^{(1)}\right)^* \cdot \underline{H}(\underline{0}, \underline{M}^{(1)})\, ds\right\} \tag{4.9}$$

where "Re" denotes the real part, "*" denotes the complex conjugate, and $\underline{M}^{(1)}$ is given by eq. (2.11) of [1] which is expressed in abbreviated notation as

$$\underline{M}^{(1)} = \sum_{i=1}^{K1} V_i \underline{M}_i \,. \tag{4.10}$$

In (4.9), the subscript "a" stands for "alternative". Substitution of (4.10) into (4.9) gives

$$P_{ta}^{(1)} = \mathrm{Re}\left\{\sum_{i=1}^{K1} Y_{ii}^{rec} \left|V_i\right|^2\right\} \tag{4.11}$$

where Y_{ii}^{rec} is, with reference to eq. (2.25) of [1], the ii^{th} element of the matrix

$$\begin{bmatrix} Y^{1,1TM,1TM} & Y^{1,1TM,1TE} \\ Y^{1,1TE,1TM} & Y^{1,1TE,1TE} \end{bmatrix}.$$

The above matrix is diagonal. The subscript "rec" in (4.11) stands for "rectangular waveguide". Equation (4.11) is recast as

$$P_{ta}^{(1)} = -\mathrm{Im}\left\{\sum_{i=1}^{K1} (-j\eta Y_{ii}^{rec}) \left|\frac{V_i}{\sqrt{\eta}}\right|^2\right\} \tag{4.12}$$

where "Im" denotes the imaginary part. Numerically, $P_{ta}^{(1)}$ of (4.12) should be equal to $P_t^{(1)}$ of (4.7).

An alternative expression for the time-average power radiated by the magnetic current $\underline{M}^{(2)}$ in the right-hand rectangular waveguide of Fig. 2 is $P_{ta}^{(2)}$ given by

$$P_{ta}^{(2)} = -\mathrm{Re}\left\{\iint_{A_2} \left(\underline{M}^{(2)}\right)^* \cdot \underline{H}(\underline{0}, \underline{M}^{(2)})\, ds\right\} \tag{4.13}$$

where $\underline{M}^{(2)}$ is given by eq. (2.12) of [1] which is expressed in abbreviated form as

$$\underline{M}^{(2)} = \sum_{i=K1+1}^{K2} V_i \underline{M}_i \tag{4.14}$$

where, as in the main program,

$$K2 = 2 * K1\,. \tag{4.15}$$

Substitution of (4.14) into (4.13) gives

$$P_{ta}^{(2)} = \mathrm{Re}\left\{\sum_{i=K1+1}^{K2} Y_{ii}^{rec} |V_i|^2\right\} \tag{4.16}$$

where Y_{ii}^{rec} is, with reference to eq. (2.27) of [1], the $(i-K1,\, i-K1)^{th}$ element of the matrix

$$\begin{bmatrix} \mathrm{Y2.2TM.2TM} & \mathrm{Y2.2TM.2TE} \\ \mathrm{Y2.2TE.2TM} & \mathrm{Y2.2TE.2TE} \end{bmatrix}.$$

The above matrix is diagonal. Equation (4.16) is recast as

$$P_{ta}^{(2)} = -\mathrm{Im}\left\{\sum_{i=K1+1}^{K2} (-j\eta Y_{ii}^{rec}) \left|\frac{V_i}{\sqrt{\eta}}\right|^2\right\}. \tag{4.17}$$

Numerically, $P_{ta}^{(2)}$ of (4.16) should be l to $P_t^{(2)}$ of (4.8).

4.4 Reduction of the C's of (4.3)–(4.6) to Those of [2] When $B_{01}^{\mathrm{TM}e} = 1$ and All the Other B's Are Zero

When $B_{01}^{\mathrm{TM}e} = 1$ and all the other B's are zero, the excitation employed in the present report is, as stated in Section 3.5, that in [2] multiplied by $e^{j\beta_{01}^{\mathrm{TM}} L_3}/\sqrt{Z_{01}^{\mathrm{TM}eo}}$. In this case, the V's in (4.3)–(4.6) are those of [2] multiplied by $e^{j\beta_{01}^{\mathrm{TM}} L_3}/\sqrt{Z_{01}^{\mathrm{TM}eo}}$. In view of (3.27), this multiplication factor is $e^{j\beta_{01}^{\mathrm{TM}} L_3}\sqrt{k/(\eta\beta_{01}^{\mathrm{TM}})}$. It follows that the C's of (4.3)–(4.6) are equal to those of eqs. (5.32), (5.33), (5.36), and (5.37) of [2] when $B_{01}^{\mathrm{TM}e} = 1$ and all the other B's are equal to zero.

Chapter 5

The Propagating Modes in the Circular Waveguide

If several modes propagate in the circular waveguide, then eq. (6.73) of [2] has to be changed because this equation was written assuming that only the even TM_{01} and even and odd TE_{11} modes propagate and that the excitation is the even TM_{01} wave whose electric and magnetic fields are, respectively, $\underline{E}_{01}^{TMe+}$ and $\underline{H}_{01}^{TMe+}$ given by eqs. (5.1) and (5.2) of [1].[†]

5.1 Derivation of an Expression for the Electric Field of the Propagating Modes

From eqs. (B.26), (B.27), (B.30), and (B.33) of [1] for odd TM modes and from eqs. (B.55), (B.56), (B.59), and (B.62) of [1] for odd TE modes, we have

$$\underline{E}_{0s}^{TMo+} = \underline{E}_{0s}^{TMo-} = \underline{E}_{0s}^{TEo+} = \underline{E}_{0s}^{TEo-} = \underline{0}, \quad s = 1, 2, \cdots. \tag{5.1}$$

Actually, (5.1) defines the above $\underline{E}_{os}$'s because these $\underline{E}_{os}$'s were undefined before (5.1) was written. From eqs. (3.57) and (3.58) of [2],

$$\gamma_{rs}^{TM} a = j\beta_{rs}^{TM} a, \quad x_{rs} < ka \tag{5.2}$$

[†]The left-hand side of eq. (5.1) of [1] should be $\underline{E}_{01}^{TMe+}$. In an early version of [1], it was mistakenly written as $\underline{E}_{01}^{TEe+}$.

$$\gamma_{rs}^{\mathrm{TE}} a = j\beta_{rs}^{\mathrm{TE}} a, \;\; x'_{rs} < ka \tag{5.3}$$

where

$$\beta_{rs}^{\mathrm{TM}} a = \sqrt{(ka)^2 - x_{rs}^2} \tag{5.4}$$

$$\beta_{rs}^{\mathrm{TE}} a = \sqrt{(ka)^2 - x_{rs}'^2} \, . \tag{5.5}$$

In view of eqs. (5.1)–(5.3), substitution of eq. (6.55) of [2] and (3.7) into eq. (6.1) of (2) gives

$$\begin{aligned} E^{(3)} = &\sum_{r=0}\sum_{s=1} \frac{\left(B_{rs}^{\mathrm{TMe}} E_{rs}^{\mathrm{TMe}+} e^{j\beta_{rs}^{\mathrm{TM}} L_3} - C_{rs}^{\mathrm{TMe}} E_{rs}^{\mathrm{TMe}-} e^{-j\beta_{rs}^{\mathrm{TM}} L_3}\right)}{\sqrt{Z_{rs}^{\mathrm{TMeo}}}} \\ &+ \sum_{r=1}\sum_{s=1} \frac{\left(B_{rs}^{\mathrm{TMo}} E_{rs}^{\mathrm{TMo}+} e^{j\beta_{rs}^{\mathrm{TM}} L_3} - C_{rs}^{\mathrm{TMo}} E_{rs}^{\mathrm{TMo}-} e^{-j\beta_{rs}^{\mathrm{TM}} L_3}\right)}{\sqrt{Z_{rs}^{\mathrm{TMeo}}}} \\ &+ \sum_{r=0}\sum_{s=1} \frac{\left(B_{rs}^{\mathrm{TEe}} E_{rs}^{\mathrm{TEe}+} e^{j\beta_{rs}^{\mathrm{TE}} L_3} + C_{rs}^{\mathrm{TEe}} E_{rs}^{\mathrm{TEe}-} e^{-j\beta_{rs}^{\mathrm{TE}} L_3}\right)}{\sqrt{Y_{rs}^{\mathrm{TEeo}}}} \\ &+ \sum_{r=1}\sum_{s=1} \frac{\left(B_{rs}^{\mathrm{TEo}} E_{rs}^{\mathrm{TEo}+} e^{j\beta_{rs}^{\mathrm{TE}} L_3} + C_{rs}^{\mathrm{TEo}} E_{rs}^{\mathrm{TEo}-} e^{-j\beta_{rs}^{\mathrm{TE}} L_3}\right)}{\sqrt{Y_{rs}^{\mathrm{TEeo}}}} \\ &+ \text{evanescent waves} \end{aligned} \tag{5.6}$$

where

$$C_{rs}^{\mathrm{TMe}} = -B_{rs}^{\mathrm{TMe}} - 2\sqrt{\left(\frac{2\pi b}{c}\right)\left(\frac{\epsilon_r}{2}\right)}\left(\frac{S_{rs}^{\mathrm{TMe}}}{\sqrt{Z_{rs}^{\mathrm{TMeo}}}}\right) \tag{5.7}$$

$$C_{rs}^{\mathrm{TMo}} = -B_{rs}^{\mathrm{TMo}} - 2\sqrt{\left(\frac{2\pi b}{c}\right)\left(\frac{\epsilon_r}{2}\right)}\left(\frac{S_{rs}^{\mathrm{TMo}}}{\sqrt{Z_{rs}^{\mathrm{TMeo}}}}\right) \tag{5.8}$$

$$C_{rs}^{\mathrm{TEe}} = -B_{rs}^{\mathrm{TEe}} + 2\sqrt{\left(\frac{2\pi b}{c}\right)\left(\frac{\epsilon_r}{2}\right)}\left(S_{rs}^{\mathrm{TEe}}\sqrt{Y_{rs}^{\mathrm{TEeo}}}\right) \tag{5.9}$$

$$C_{rs}^{\mathrm{TEo}} = -B_{rs}^{\mathrm{TEo}} + 2\sqrt{\left(\frac{2\pi b}{c}\right)\left(\frac{\epsilon_r}{2}\right)}\left(S_{rs}^{\mathrm{TEo}}\sqrt{Y_{rs}^{\mathrm{TEoo}}}\right). \tag{5.10}$$

The first double summation in (5.6) is over all nonnegative integers r and all positive integers s such that

$$x_{rs} < ka. \tag{5.11}$$

The second double summation in (5.6) is over all positive integers r and s such that (5.11) holds. The third double summation in (5.6) is over all nonnegative integers r and all positive integers s such that

$$x'_{rs} < ka. \tag{5.12}$$

The fourth double summation in (5.6) is over all positive integers r and s such that (5.12) holds. The evanescent waves in (5.6) are those of all the evanescent modes in the circular waveguide. These modes consist of all the even and odd TM_{rs} modes for which $x_{rs} \geq ka$ and all the even and odd TE_{rs} modes for which $x'_{rs} \geq ka$.

The transverse part of the $\underline{E}_{rs}^{\mathrm{TMe+}}$ term in (5.6) reduces to $\sqrt{Z_{rs}^{\mathrm{TMeo}}}\underline{e}_{rs}^{\mathrm{TMe}}$ when $B_{rs}^{\mathrm{TMe}} = 1$ and $z = L_3$. This reduced term carries unit time-average power. The multiplier of $C_{rs}^{\mathrm{TMe}}\underline{E}_{rs}^{\mathrm{TMe-}}$ in (5.6) was chosen such that the part of C_{rs}^{TMe} attributable to direct reflection of the incident even TM_{rs} wave[†] about $z = L_3$ is -1 when $B_{rs}^{\mathrm{TMe}} = 1$. Similar remarks apply to the $\underline{E}_{rs}^{\mathrm{TMo+}}$ and $\underline{E}_{rs}^{\mathrm{TMo-}}$ terms in (5.6).

The $\underline{E}_{rs}^{\mathrm{TEe+}}$ term in (5.6) reduces to $\underline{e}_{rs}^{\mathrm{TEe}}/\sqrt{Y_{rs}^{\mathrm{TEeo}}}$ when $B_{rs}^{\mathrm{TEe}} = 1$ and $z = L_3$. This reduced term carries unit time-average power. The multiplier of $C_{rs}^{\mathrm{TEe}}\underline{E}_{rs}^{\mathrm{TEe-}}$ in (5.6) was chosen such that the part of C_{rs}^{TEe} attributable to direct reflection of the incident even TE_{rs} wave[‡] about $z = L_3$ is -1 when $B_{rs}^{\mathrm{TEe}} = 1$. Similar remarks apply to the $\underline{E}_{rs}^{\mathrm{TEo+}}$ and the $\underline{E}_{rs}^{\mathrm{TEo-}}$ terms in (5.6).

5.2 The Coefficients C_{rs}^{TMe} and C_{rs}^{TMo}

In this section, C_{rs}^{TMe} and C_{rs}^{TMo} of (5.7) and (5.8) are expressed in a form

[†]The electric field of this incident wave is the $\underline{E}_{rs}^{\mathrm{TMe+}}$ term in (5.6).

[‡]The electric field of this incident wave is the $\underline{E}_{rs}^{\mathrm{TEe+}}$ term in (5.6).

suitable for calculation. In (5.7) and (5.8), $S_{rs}^{\mathrm{TM}e}$ and $S_{rs}^{\mathrm{TM}o}$ are given by eqs. (6.56) and (6.57) of [2]:

$$S_{rs}^{\mathrm{TM}e} = -\sqrt{\eta}\sum_{\gamma=1}^{2}(-1)^{r\gamma}\sum_{q=0}\left(\frac{z^{\mathrm{TM}2}}{c}\right)\sum_{\substack{p=0\\p+q\neq 0}}\epsilon_{pq}C_1\phi_p^{b2} \tag{5.13}$$

$$S_{rs}^{\mathrm{TM}o} = -\sqrt{\eta}\sum_{\gamma=1}^{2}(-1)^{(r+1)\gamma}\sum_{q=0}\left(\frac{z^{\mathrm{TM}2}}{c}\right)\sum_{\substack{p=0\\p+q\neq 0}}\epsilon_{pq}C_1\phi_p^{b1} \tag{5.14}$$

where, because of the additional factors $\sqrt{\eta}$ and $(-1)^{r\gamma}$ in (5.13) and (5.14), C_1 is given not by (6.61) of [2] but by

$$C_1 = q\left(\frac{V_{pq}^{\gamma\mathrm{TM}}}{\sqrt{\eta}}\right) + \left(\frac{pc}{b}\right)\left(\frac{V_{pq}^{\gamma\mathrm{TE}}}{\sqrt{\eta}}\right). \tag{5.15}$$

The factor of $\sqrt{\eta}$ explicit in (5.13) and (5.14) will cancel out when $S_{rs}^{\mathrm{TM}e}$ and $S_{rs}^{\mathrm{TM}o}$ are substituted into (5.7) and (5.8). Equations (6.28) and (6.29) of [2] are

$$\phi_p^{b1} = \phi_p^{(1)}\cos\left(\frac{rb}{2x_o}\right) - \phi_p^{(2)}\sin\left(\frac{rb}{2x_o}\right) \tag{5.16}$$

$$\phi_p^{b2} = \phi_p^{(2)}\cos\left(\frac{rb}{2x_o}\right) + \phi_p^{(1)}\sin\left(\frac{rb}{2x_o}\right). \tag{5.17}$$

Comparing eqs. (6.15) and (3.76) of [2], we see that, since $x_{rs} < ka$,

$$\frac{z^{\delta 2}}{c} = \tfrac{1}{2}\hat{G}_q^{\delta}, \quad \delta = \mathrm{TM}, \mathrm{TE}. \tag{5.18}$$

Substitution of (5.18) into (5.13) and (5.14) gives

$$S_{rs}^{\mathrm{TM}e} = -\frac{\sqrt{\eta}}{2}\sum_{\gamma=1}^{2}(-1)^{r\gamma}\sum_{q=0}\hat{G}_q^{\mathrm{TM}}\sum_{\substack{p=0\\p+q\neq 0}}\epsilon_{pq}\phi_p^{b2}C_1 \tag{5.19}$$

$$S_{rs}^{\mathrm{TM}o} = -\frac{\sqrt{\eta}}{2}\sum_{\gamma=1}^{2}(-1)^{(r+1)\gamma}\sum_{q=0}\hat{G}_{.}^{\mathrm{TM}}\sum_{\substack{p=0\\p+q\neq 0}}\epsilon_{pq}\phi_p^{b1}C_1. \tag{5.20}$$

Equation (5.19) is similar to eq. (6.86) of [2].†

Substituting (3.27) and (5.19) into (5.7), we obtain, upon use of (5.15),

$$C_{rs}^{\mathrm{TM}e} = -B_{rs}^{\mathrm{TM}e} + \frac{1}{2}\sum_{\gamma=1}^{2}\sum_{q=0}\sum_{\substack{p=0\\p+q\neq0}}\left\{I_j^{\gamma\mathrm{TM},\mathrm{TM}e}\left(\frac{V_{pq}^{\gamma\mathrm{TM}}}{\sqrt{\eta}}\right) + I_j^{\gamma\mathrm{TE},\mathrm{TM}e}\left(\frac{V_{pq}^{\gamma\mathrm{TE}}}{\sqrt{\eta}}\right)\right\} \tag{5.21}$$

where, in view of eq. (6.60) of [2] and (3.110), $I_j^{\gamma\mathrm{TM},\mathrm{TM}e}$ and $I_j^{\gamma\mathrm{TE},\mathrm{TM}e}$ are given by the right-hand sides of (3.57), (3.58), (3.63), and (3.64) with (m,n,i) replaced by (p,q,j). Substitution of (3.27) and (5.20) into (5.8) gives, upon use of (5.15),

$$C_{rs}^{\mathrm{TM}o} = -B_{rs}^{\mathrm{TM}o} + \frac{1}{2}\sum_{\gamma=1}^{2}\sum_{q=0}\sum_{\substack{p=0\\p+q\neq0}}\left\{I_j^{\gamma\mathrm{TM},\mathrm{TM}o}\left(\frac{V_{pq}^{\gamma\mathrm{TM}}}{\sqrt{\eta}}\right) + I_j^{\gamma\mathrm{TE},\mathrm{TM}o}\left(\frac{V_{pq}^{\gamma\mathrm{TE}}}{\sqrt{\eta}}\right)\right\} \tag{5.22}$$

where, in view of (3.111), $I_j^{\gamma\mathrm{TM},\mathrm{TM}o}$ and $I_j^{\gamma\mathrm{TE},\mathrm{TM}o}$ are given by the right-hand sides of (3.65), (3.66), (3.69), and (3.70) with (m,n,i) replaced by (p,q,j).

5.3 The Coefficients $C_{rs}^{\mathrm{TE}e}$ and $C_{rs}^{\mathrm{TE}o}$

In this section, $C_{rs}^{\mathrm{TE}e}$ and $C_{rs}^{\mathrm{TE}o}$ of (5.9) and (5.10) are expressed in a form suitable for calculation. In (5.9) and (5.10), $S_{rs}^{\mathrm{TE}e}$ and $S_{rs}^{\mathrm{TE}o}$ are given by eqs. (6.58) and (6.59) of [2] with $\gamma_{rs}^{\mathrm{TE}}$ replaced by $j\beta_{rs}^{\mathrm{TE}}$:

$$S_{rs}^{\mathrm{TE}e} = \frac{\sqrt{\eta}}{\sqrt{x_{rs}'^2 - r^2}}\sum_{\gamma=1}^{2}(-1)^{(r+1)\gamma}\sum_{q=0}\sum_{\substack{p=0\\p+q\neq0}}\epsilon_{pq}$$
$$\cdot\left\{r\left(\frac{z^{\mathrm{TE}2}}{c}\right)C_1\phi_p^{b1} - \left(\frac{\sin\phi_o}{\phi_o}\right)\left(\frac{z^{\mathrm{TE}4}}{c}\right)\left(\frac{x_{rs}'^2 C_2\phi_p^{b4}}{j\beta_{rs}^{\mathrm{TE}}a}\right)\right\} \tag{5.23}$$

†There is an error in eq. (6.86) of [2]. The right-hand side of this equation should be multiplied by $-1/2$.

$$S_{rs}^{\mathrm{TE}o} = \frac{-\sqrt{\eta}}{\sqrt{x_{rs}'^2 - r^2}} \sum_{\gamma=1}^{2} (-1)^{\gamma r} \sum_{q=0} \sum_{\substack{p=0 \\ p+q\neq 0}} \epsilon_{pq}$$

$$\cdot \left\{ r\left(\frac{z^{\mathrm{TE}2}}{c}\right) C_1 \phi_p^{b2} + \left(\frac{\sin\phi_o}{\phi_o}\right)\left(\frac{z^{\mathrm{TE}4}}{c}\right)\left(\frac{x_{rs}'^2 C_2 \phi_p^{b3}}{j\beta_{rs}^{\mathrm{TE}} a}\right)\right\} \quad (5.24)$$

where, because of the additional factors $\sqrt{\eta}$ and $(-1)^{\gamma r}$ in (5.23) and (5.24), C_1 and C_2 are given not by eqs. (6.61) and (6.62) of [2] but by (5.15) and

$$C_2 = \left(\frac{pc}{b}\right)\left(\frac{V_{pq}^{\gamma\mathrm{TM}}}{\sqrt{\eta}}\right) - q\left(\frac{V_{pq}^{\gamma\mathrm{TE}}}{\sqrt{\eta}}\right). \quad (5.25)$$

Equations (6.30) and (6.31) of [2] are

$$\phi_p^{b3} = \phi_p^{(3)} \cos\left(\frac{rb}{2x_o}\right) - \phi_p^{(4)} \sin\left(\frac{rb}{2x_o}\right) \quad (5.26)$$

$$\phi_p^{b4} = \phi_p^{(4)} \cos\left(\frac{rb}{2x_o}\right) + \phi_p^{(3)} \sin\left(\frac{rb}{2x_o}\right). \quad (5.27)$$

Comparing eqs. (6.16) and (3.108) of [2], we see that

$$\frac{z^{\mathrm{TE}4}}{c} = -\frac{j}{2}\hat{G}_q^{(4)}. \quad (5.28)$$

Substitution of (5.18) and (5.28) into (5.23) and (5.24) gives

$$S_{rs}^{\mathrm{TE}e} = \frac{\sqrt{\eta}}{2\sqrt{x_{rs}'^2 - r^2}} \sum_{\gamma=1}^{2} (-1)^{(r+1)\gamma} \sum_{q=0} \sum_{\substack{p=0 \\ p+q\neq 0}} \epsilon_{pq}$$

$$\cdot \left\{ r\hat{G}_q^{\mathrm{TE}} \phi_p^{b1} C_1 + \left(\frac{\sin\phi_o}{\phi_o}\right)\left(\frac{\hat{G}_q^{(4)} x_{rs}'^2 \phi_p^{b4}}{\beta_{rs}^{\mathrm{TE}} a}\right) C_2 \right\} \quad (5.29)$$

$$S_{rs}^{\mathrm{TE}o} = \frac{\sqrt{\eta}}{2\sqrt{x_{rs}'^2 - r^2}} \sum_{\gamma=1}^{2} (-1)^{\gamma r} \sum_{q=0} \sum_{\substack{p=0 \\ p+q\neq 0}} \epsilon_{pq}$$

$$\cdot \left\{ -r\hat{G}_q^{\mathrm{TE}} \phi_p^{b2} C_1 + \left(\frac{\sin\phi_o}{\phi_o}\right)\left(\frac{\hat{G}_q^{(4)} x_{rs}'^2 \phi_p^{b3}}{\beta_{rs}^{\mathrm{TE}} a}\right) C_2 \right\}. \quad (5.30)$$

Equations (5.29) and (5.30) are generalizations of eqs. (6.93) and (6.94) of [2].†

Substituting (3.30) and (5.29) into (5.9), we obtain, upon use of (5.15), (5.25), and eq. (6.60) of [2],

$$C_{rs}^{\mathrm{TEe}} = -B_{rs}^{\mathrm{TEe}} + \tfrac{1}{2}\sum_{\gamma=1}^{2}\sum_{q=0}\sum_{\substack{p=0\\p+q\neq 0}}\left\{ I_j^{\gamma\mathrm{TM},\mathrm{TEe}}\left(\frac{V_{pq}^{\gamma\mathrm{TM}}}{\sqrt{\eta}}\right) + I_j^{\gamma\mathrm{TE},\mathrm{TEe}}\left(\frac{V_{pq}^{\gamma\mathrm{TE}}}{\sqrt{\eta}}\right)\right\} \tag{5.31}$$

where $I_j^{\gamma\mathrm{TM},\mathrm{TEe}}$ is given by the right-hand sides of (3.71) and (3.79) with (m,n,i) replaced by (p,q,j) and $I_j^{\gamma\mathrm{TE},\mathrm{TEe}}$ is given by the right-hand sides of (3.72) and (3.80) with (m,n,i) replaced by (p,q,j). On these right-hand sides, $I_{i,rs}^{\mathrm{TEe}\phi}$ and $I_{i,rs}^{\mathrm{TEe}z}$ are, before replacement of (m,n,i) by (p,q,j), given by (3.112) and (3.113).

Substitution of (3.30) and (5.30) into (5.10) gives, upon use of (5.15) and (5.25),

$$C_{rs}^{\mathrm{TEo}} = -B_{rs}^{\mathrm{TEo}} + \tfrac{1}{2}\sum_{\gamma=1}^{2}\sum_{q=0}\sum_{\substack{p=0\\p+q\neq 0}}\left\{ I_j^{\gamma\mathrm{TM},\mathrm{TEo}}\left(\frac{V_{pq}^{\gamma\mathrm{TM}}}{\sqrt{\eta}}\right) + I_j^{\gamma\mathrm{TE},\mathrm{TEo}}\left(\frac{V_{pq}^{\gamma\mathrm{TE}}}{\sqrt{\eta}}\right)\right\}. \tag{5.32}$$

In (5.32), $I_j^{\gamma\mathrm{TM},\mathrm{TEo}}$ is given by the right-hand sides of (3.81) and (3.86) with (m,n,i) replaced by (p,q,j) and $I_j^{\gamma\mathrm{TE},\mathrm{TEo}}$ is given by the right-hand sides of (3.82) and (3.87) with (m,n,i) replaced by (p,q,j). On these right-hand sides, $I_{i,rs}^{\mathrm{TEo}\phi}$ and $I_{i,rs}^{\mathrm{TEo}z}$ are, before replacement of (m,n,i) by (p,q,j), given by (3.114) and (3.115).

†Equation (6.94) of [2] is not correct. Please correct eq. (6.94) of [2] by replacing $\sum_{\gamma=1}^{2}$ therein by $\sum_{\gamma=1}^{2}(-1)^{\gamma}$.

5.4 The Time-Average Power of the Propagating Modes Produced by the Magnetic Current in the Circular Waveguide

The electric field in the circular waveguide of Fig. 2 is produced by the sources $\underline{J}^{\text{imp}}$, $-\underline{M}^{(1)}$ and $-\underline{M}^{(2)}$ therein. The partial electric field produced by the magnetic currents $-\underline{M}^{(1)}$ and $-\underline{M}^{(2)}$ is $\underline{E}^{(3)}(\underline{0}, -\underline{M}^{(1)} - \underline{M}^{(2)})$ given by

$$
\begin{aligned}
\underline{E}^{(3)}(\underline{0}, -\underline{M}^{(1)} - \underline{M}^{(2)}) = &-\sum_{r=0}\sum_{s=1} \frac{(C_{rs}^{\text{TM}e} + B_{rs}^{\text{TM}e})\underline{E}_{rs}^{\text{TM}e-} e^{-j\beta_{rs}^{\text{TM}} L_3}}{\sqrt{Z_{rs}^{\text{TM}eo}}} \\
&-\sum_{r=1}\sum_{s=1} \frac{(C_{rs}^{\text{TM}o} + B_{rs}^{\text{TM}o})\underline{E}_{rs}^{\text{TM}o-} e^{-j\beta_{rs}^{\text{TM}} L_3}}{\sqrt{Z_{rs}^{\text{TM}eo}}} \\
&+\sum_{r=0}\sum_{s=1} \frac{(C_{rs}^{\text{TE}e} + B_{rs}^{\text{TE}e})\underline{E}_{rs}^{\text{TE}e-} e^{-j\beta_{rs}^{\text{TE}} L_3}}{\sqrt{Y_{rs}^{\text{TE}eo}}} \\
&+\sum_{r=1}\sum_{s=1} \frac{(C_{rs}^{\text{TE}o} + B_{rs}^{\text{TE}o})\underline{E}_{rs}^{\text{TE}o-} e^{-j\beta_{rs}^{\text{TE}} L_3}}{\sqrt{Y_{rs}^{\text{TE}eo}}} \\
&+\text{evanescent waves.}
\end{aligned} \tag{5.33}
$$

Equation (5.33) was obtained by removing from $\underline{E}^{(3)}$ of (5.6) the electric field produced by $\underline{J}^{\text{imp}}$. The latter electric field consists of the double summations on the right-hand side of (5.6) with $C_{rs}^{\text{TM}e}$, $C_{rs}^{\text{TM}o}$, $C_{rs}^{\text{TE}e}$, and $C_{rs}^{\text{TE}eo}$ replaced by $-B_{rs}^{\text{TM}e}$, $-B_{rs}^{\text{TM}o}$, $-B_{rs}^{\text{TE}e}$, and $-B_{rs}^{\text{TE}eo}$, respectively.† As in (5.6), the first double summation in (5.33) is over all nonnegative integers r and all positive integers s such that (5.11) holds. The second double summation in (5.33) is over all positive integers r and s such that (5.11) holds. The third double summation in (5.33) is over all nonnegative integers r and all positive integers s such that (5.12) holds. The fourth double summation in (5.33) is over all positive integers r and s such that (5.12) holds.

†If $-\underline{M}^{(1)}$ and $-\underline{M}^{(2)}$ were zero, the $S_{rs}^{\text{TM}e}$, $S_{rs}^{\text{TM}o}$, $S_{rs}^{\text{TE}e}$, and $S_{rs}^{\text{TE}o}$ terms in (5.7)–(5.10) would be zero so that the C's would reduce to the negatives of the B's.

The time-average power of $\underline{E}^{(3)}(\underline{0}, -\underline{M}^{(1)}, -\underline{M}^{(2)})$ of (5.33) is P_{rm} given by

$$P_{rm} = \sum_{r=0}\sum_{s=1}\left|C_{rs}^{\mathrm{TMe}} + B_{rs}^{\mathrm{TMe}}\right|^2 + \sum_{r=1}\sum_{s=1}\left|C_{rs}^{\mathrm{TMo}} + B_{rs}^{\mathrm{TMo}}\right|^2$$
$$+ \sum_{r=0}\sum_{s=1}\left|C_{rs}^{\mathrm{TEe}} + B_{rs}^{\mathrm{TEe}}\right|^2 + \sum_{r=1}\sum_{s=1}\left|C_{rs}^{\mathrm{TEo}} + B_{rs}^{\mathrm{TEo}}\right|^2 . \quad (5.34)$$

The quantity P_{rm} of (5.34) is the time-average power of the propagating modes produced by the magnetic currents $-\underline{M}^{(1)}$ and $-\underline{M}^{(2)}$ in the circular waveguide. In brief, P_{rm} of (5.34) is the time-average power radiated by the magnetic current in the circular waveguide.†

5.5 The Time-Average Power Required to Maintain the Magnetic Current in the Circular Waveguide

The time-average power required to maintain the magnetic current in the circular waveguide is P_{rma} given by

$$P_{rma} = -\mathrm{Re}\left\{\iint_S \underline{M}^* \cdot \underline{H}(\underline{0}, \underline{M})\, ds\right\} \quad (5.35)$$

where $\underline{M}$ represents the combination of the magnetic currents $-\underline{M}^{(1)}$ and $-\underline{M}^{(2)}$ in the circular waveguide, $\underline{H}(\underline{0}, \underline{M})$ is the magnetic field due to this combination, and S represents the surfaces on which $\underline{M}$ exists. Now, P_{rma} can be viewed as an alternative expression for the time-average power radiated by the magnetic current in the circular waveguide.‡ With $\underline{M}^{(1)}$ and $\underline{M}^{(2)}$ given by (4.10) and (4.14), (5.35) becomes

$$P_{rma} = \mathrm{Re}\left\{\tilde{V}^* Y^3 \vec{V}\right\} \quad (5.36)$$

where $\vec{V}$ is the column vector of the V's in (4.10) and (4.14) and Y^3 is the square matrix given by eq. (2.29) of [1].

†The subscript "rm" on P_{rm} stands for "radiated by the magnetic current".

‡The subscript "a" on P_{rma} stands for alternative.

Equation (2.22) of [1] is written concisely as

$$[Y^1 + Y^2 + Y^3]\vec{V} = \vec{I}. \tag{5.37}$$

From (5.37),

$$Y^3\vec{V} = \vec{I} - [Y^1 + Y^2]\vec{V}. \tag{5.38}$$

Premultiplying (5.38) by $\tilde{V}^*$ and taking the real part, we have

$$\mathrm{Re}\left\{\tilde{V}^* Y^3 \vec{V}\right\} = \mathrm{Re}\left\{\tilde{V}^* \vec{I}\right\} - \mathrm{Re}\left\{\tilde{V}^*[Y^1 + Y^2]\vec{V}\right\}. \tag{5.39}$$

Now,

$$\tilde{V}^*[Y^1 + Y^2]\vec{V} = \sum_{i=1}^{\mathrm{N1}} Y_{ii}^{\mathrm{rec}} |V_i|^2 + \sum_{i=1}^{\mathrm{N1}} Y_{i+\mathrm{N1},i+\mathrm{N1}}^{\mathrm{rec}} |V_{i+\mathrm{N1}}|^2 \tag{5.40}$$

where Y_{ii}^{rec} and $Y_{i+\mathrm{N1},i+\mathrm{N1}}^{\mathrm{rec}}$ are the same as in (4.11) and (4.15). Taking the real part of (5.40) and using (4.11) and (4.15), we obtain

$$\mathrm{Re}\left\{\tilde{V}^*[Y^1 + Y^2]\vec{V}\right\} = P_{ta}^{(1)} + P_{ta}^{(2)}. \tag{5.41}$$

Substituting (5.41) into (5.39) and using (5.36), we obtain

$$P_{rma} = \mathrm{Re}\left\{\tilde{V}^* \vec{I}\right\} - P_{ta}^{(1)} - P_{ta}^{(2)} \tag{5.42}$$

which is recast as

$$P_{rma} = -\mathrm{Im}\left\{\left[\frac{\tilde{V}^*}{\sqrt{\eta}}\right]\left[-j\sqrt{\eta}\vec{I}\right]\right\} - P_{ta}^{(1)} - P_{ta}^{(2)}. \tag{5.43}$$

Numerically, P_{rma} of (5.43) should be equal to P_{rm} of (5.34).

5.6 The Time-Average Power of the Propagating Modes in the Circular Waveguide

The time-average power of the z-traveling (incident) waves in expression (5.6) for the electric field in the circular waveguide is P_{inc} given by

$$P_{inc} = \sum_{r=0}\sum_{s=1}\left|B_{rs}^{\mathrm{TM}e}\right|^2 + \sum_{r=1}\sum_{s=1}\left|B_{rs}^{\mathrm{TM}o}\right|^2$$

$$+ \sum_{r=0}\sum_{s=1}\left|B_{rs}^{\mathrm{TE}e}\right|^2 + \sum_{r=1}\sum_{s=1}\left|B_{rs}^{\mathrm{TE}o}\right|^2. \tag{5.44}$$

The subscript "inc" on P_{inc} stands for "incident". The time-average power of the $-z$-traveling waves in expression (5.6) is P_r given by

$$P_r = \sum_{r=0}\sum_{s=1}\left|C_{rs}^{\mathrm{TMe}}\right|^2 + \sum_{r=1}\sum_{s=1}\left|C_{rs}^{\mathrm{TMo}}\right|^2$$
$$+\sum_{r=0}\sum_{s=1}\left|C_{rs}^{\mathrm{TEe}}\right|^2 + \sum_{r=1}\sum_{s=1}\left|C_{rs}^{\mathrm{TEo}}\right|^2 . \tag{5.45}$$

The subscript "r" on P_r stands for "reflected". Numerically, the time-average power P_{inc} of the z-traveling waves in the circular waveguide must equal the sum of the time-average power P_r reflected in the circular waveguide and the time-average power $P_t^{(1)} + P_t^{(2)}$ transmitted into the rectangular waveguides:

$$P_{inc} = P_r + P_t^{(1)} + P_t^{(2)}. \tag{5.46}$$

According to (5.46), part of P_{inc} is reflected in the circular waveguide, and the rest of P_{inc} is transmitted into the rectangular waveguides.

5.7 Comparison of the Coefficients C_{01}^{TMe}, C_{11}^{TEe}, and C_{11}^{TEo} with Those of [2]

The excitation in [2] is the z-traveling wave whose electric field is $\mathbf{E}_{01}^{\mathrm{TMe}+}$ given by eq. (5.1) of [1].[†] This excitation gives rise to the electric field $\mathbf{E}^{(3)}$ in the circular waveguide. Assuming, as in [2], that only the TM_{01} and TE_{11} modes can propagate in the circular waveguide, eq. (6.75) of [2] gives

$$\left(\frac{e^{j\beta_{01}^{\mathrm{TM}}L_3}}{\sqrt{Z_{01}^{\mathrm{TMeo}}}}\right)\mathbf{E}^{(3)} = \left(\frac{e^{j\beta_{01}^{\mathrm{TM}}L_3}}{\sqrt{Z_{01}^{\mathrm{TMeo}}}}\right)\mathbf{E}_{01}^{\mathrm{TMe}+} - \left(\frac{C_{01}^{\mathrm{TMe}}e^{-j\beta_{01}^{\mathrm{TM}}L_3}}{\sqrt{Z_{01}^{\mathrm{TMeo}}}}\right)\mathbf{E}_{01}^{\mathrm{TMe}-}$$
$$+\left(\frac{e^{-j\beta_{11}^{\mathrm{TE}}L_3}}{\sqrt{Z_{11}^{\mathrm{TEeo}}}}\right)\left(C_{11}^{\mathrm{TEe}}\mathbf{E}_{11}^{\mathrm{TEe}-} + C_{11}^{\mathrm{TEo}}\mathbf{E}_{11}^{\mathrm{TEo}-}\right)$$
$$+\text{evanescent waves.} \tag{5.47}$$

The right-hand side of (5.47) is the electric field that would exist in the circular waveguide if the excitation in [2] were the z-traveling wave whose electric field is $\mathbf{E}_{01}^{\mathrm{TMe}+}e^{j\beta_{01}^{\mathrm{TM}}L_3}/\sqrt{Z_{01}^{\mathrm{TMeo}}}$.

[†]The excitation in [2] is the same as that in [1]. The left-hand side of eq. (5.1) of [1] should be $\mathbf{E}_{01}^{\mathrm{TMe}+}$. In an early version of [1], it was mistakenly written as $\mathbf{E}_{01}^{\mathrm{TEe}+}$.

If $B_{01}^{\mathrm{TMe}} = 1$ is the only nonzero B in (3.1), then the excitation would be the z-traveling wave whose electric field is $\underline{E}_{01}^{\mathrm{TMe}+} e^{j\beta_{01}^{\mathrm{TM}} L_3} / \sqrt{Z_{01}^{\mathrm{TMeo}}}$, the same as in the previous sentence. If only the TM_{01} and TE_{11} modes can propagate in the circular waveguide, then this excitation would give rise to the electric field $\underline{E}^{(3)}$ of (5.6) in the circular waveguide:

$$\begin{aligned}\underline{E}^{(3)} &= \left(\frac{e^{j\beta_{01}^{\mathrm{TM}} L_3}}{\sqrt{Z_{01}^{\mathrm{TMeo}}}}\right) \underline{E}_{01}^{\mathrm{TMe}+} - \left(\frac{C_{01}^{\mathrm{TMe}} e^{-j\beta_{01}^{\mathrm{TM}} L_3}}{\sqrt{Z_{01}^{\mathrm{TMeo}}}}\right) \underline{E}_{01}^{\mathrm{TMe}-} \\ &+ \left(\frac{e^{-j\beta_{11}^{\mathrm{TE}} L_3}}{\sqrt{Y_{11}^{\mathrm{TEeo}}}}\right) \left(C_{11}^{\mathrm{TEe}} \underline{E}_{11}^{\mathrm{TEe}-} + C_{11}^{\mathrm{TEo}} \underline{E}_{11}^{\mathrm{TEo}-}\right) \\ &+ \text{evanescent waves.} \qquad (5.48)\end{aligned}$$

The right-hand sides of (5.47) and (5.48) are equal to each other because they are both the electric field produced by the same excitation—namely, the z-traveling wave whose electric field is $\underline{E}_{01}^{\mathrm{TMe}+} e^{j\beta_{01}^{\mathrm{TM}} L_3} / \sqrt{Z_{01}^{\mathrm{TMeo}}}$. Therefore, the C's in (5.48) are equal to those in (5.47). This means that if $B_{01}^{\mathrm{TMe}} = 1$ is the only nonzero B in (3.1) and if only the TM_{01} and TE_{11} modes can propagate in the circular waveguide, then the coefficients C_{01}^{TMe}, C_{11}^{TEe}, and C_{11}^{TEo} in (5.6) are, respectively, equal to those in eq. (6.75) of [2]. In Appendix B, we show that expressions (5.21), (5.31), and (5.32) for C_{01}^{TMe}, C_{11}^{TEe}, and C_{11}^{TEo} reduce to the right-hand sides of eqs. (6.88), (6.96), and (6.99) of [2] when $B_{01}^{\mathrm{TMe}} = 1$ is the only nonzero B in (3.1) and when only the TM_{01} and TE_{11} modes can propagate in the circular waveguide.

Chapter 6

The Tangential Electric Field in the Apertures

In eqs. (3.328), (3.332), (3.336), and (3.339) of [3] for the normalized ϕ- and z-components of the electric field in the apertures, $\underline{E}_{01}^{\mathrm{TMe}+}$ is the electric field of the z-traveling wave that would exist in the circular waveguide if the apertures were closed. The subscript "rms" on $\underline{E}_{01}^{\mathrm{TMe}+}$ in eqs. (3.328), (3.332), (3.336), and (3.339) of [3] denotes the root mean square value of the transverse part over the waveguide cross section at $z = 0$. In the present report, the electric field of the z-traveling waves that would exist if the apertures were closed is not $\underline{E}_{01}^{\mathrm{TMe}+}$ but $\underline{E}^{(3+)}(\underline{J}^{\mathrm{imp}}, \underline{0})$ given by (3.1):

$$\underline{E}^{(3+)}(\underline{J}^{\mathrm{imp}}, \underline{0}) = \sum_{r,s} \sum_{f=e,o} \left(\frac{B_{rs}^{\mathrm{TM}f} e^{j\beta_{rs}^{\mathrm{TM}} L_3}}{\sqrt{Z_{rs}^{\mathrm{TM}eo}}} \right) \underline{E}_{rs}^{\mathrm{TM}f+}$$

$$+ \sum_{r,s} \sum_{f=e,o} \left(\frac{B_{rs}^{\mathrm{TE}f} e^{j\beta_{rs}^{\mathrm{TE}} L_3}}{\sqrt{Y_{rs}^{\mathrm{TE}eo}}} \right) \underline{E}_{rs}^{\mathrm{TE}f+}. \tag{6.1}$$

Hence, $\left|\underline{E}_{01}^{\mathrm{TMe}+}\right|_{\mathrm{rms}}$ should be replaced by $\left|\underline{E}^{(3+)}(\underline{J}^{\mathrm{imp}}, \underline{0})\right|_{\mathrm{rms}}$ in eqs. (3.328), (3.332), (3.336), and (3.339) of [3].

By definition,

$$\left|\underline{E}^{(3+)}(\underline{J}^{\mathrm{imp}}, \underline{0})\right|_{\mathrm{rms}} = \left\{ \frac{1}{\pi a^2} \int_0^a \rho \, d\rho \int_0^{2\pi} d\phi \left[\left(\underline{E}^{(3+)}\right)_t \cdot \left(\underline{E}^{(3+)}\right)_t^* \right]_{z=0} \right\}^{\frac{1}{2}} \tag{6.2}$$

where the superscript "*" denotes the complex conjugate and the subscript "t" denotes the transverse part. On the right-hand side of (6.2), $\underline{E}^{(3+)}$ stands for $\underline{E}^{(3+)}(\underline{J}^{\text{imp}}, \underline{0})$. Substituting (6.1) into (6.2) and using eqs. (B.1), (B.26), (B.35), (B.55), and (B.64) of [1], we obtain

$$\left|\underline{E}^{(3+)}(\underline{J}^{\text{imp}}, \underline{0})\right|_{\text{rms}} = \frac{1}{\sqrt{\pi}a}\left\{\sum_{r,s}\sum_{f=e,o}\left|B_{rs}^{\text{TM}f}\right|^2 Z_{rs}^{\text{TMeo}} + \sum_{r,s}\sum_{f=e,o}\frac{\left|B_{rs}^{\text{TE}f}\right|^2}{Y_{rs}^{\text{TEeo}}}\right\}^{\frac{1}{2}}. \tag{6.3}$$

Substitution of (3.27) and (3.30) into (6.3) gives

$$\left|\underline{E}^{(3+)}(\underline{J}^{\text{imp}}, \underline{0})\right|_{\text{rms}} = \frac{\sqrt{\eta}}{\sqrt{\pi}a}\left\{\sum_{r,s}\sum_{f=e,o}\frac{\left|B_{rs}^{\text{TM}f}\right|^2 \beta_{rs}^{\text{TM}}}{k} + \sum_{r,s}\sum_{f=e,o}\frac{\left|B_{rs}^{\text{TE}f}\right|^2 k}{\beta_{rs}^{\text{TE}}}\right\}^{\frac{1}{2}}. \tag{6.4}$$

From eq. (7.11) of [2] and (6.4), we have

$$\frac{-\left|E_{01}^{\text{TMe+}}\right|_{\text{rms}}}{\left|\underline{E}^{(3+)}(\underline{J}^{\text{imp}}, \underline{0})\right|_{\text{rms}}} = \frac{-\beta_{01}^{\text{TM}}\sqrt{\eta}}{k\left\{\sum_{r,s}\sum_{f=e,o}\frac{\left|B_{rs}^{\text{TM}f}\right|^2 \beta_{rs}^{\text{TM}}}{k} + \sum_{r,s}\sum_{f=e,o}\frac{\left|B_{rs}^{\text{TE}f}\right|^2 k}{\beta_{rs}^{\text{TE}}}\right\}^{\frac{1}{2}}}. \tag{6.5}$$

Multiplying both sides of eqs. (3.328), (3.332), (3.336), and (3.339) of [3] by (6.5), we have[†]

$$\frac{E_{\phi}^{(A1)}(\phi_{\text{J}}^{(A1)}, 0)}{\left|\underline{E}^{(3+)}(\underline{J}^{\text{imp}}, \underline{0})\right|_{\text{rms}}} = -(S_a)\left(\frac{x_o}{a}\right)\sum_{q=0}^{\text{NMAX}-1}\sum_{\substack{p=0\\p+q\neq 0}}^{\text{MM}(q+1)-1}\sqrt{\frac{\epsilon_p\epsilon_q}{4}}\left(\frac{1}{k_{pq}b}\right)$$

$$\cdot\left\{\left(\frac{pa}{b}\right)\left(\frac{V_{pq}^{1\text{TM}}}{\sqrt{\eta}}\right) - \left(\frac{qa}{c}\right)\left(\frac{V_{pq}^{1\text{TE}}}{\sqrt{\eta}}\right)\right\}\sin\left(\frac{q\pi}{2}\right)\cos(p\phi_{\text{J}}^{(1)}) \tag{6.6}$$

[†]The left-hand sides of eqs (3.328), (3.332), (3.336), and (3.339) of [3] are incorrect. The correct left-hand sides are the negatives of those written in these equations (the left-hand sides of eqs. (7.12)–(7.14) of [2] are also incorrect. The correct left-hand sides are the negatives of those written in these equations). The corrected eqs. (3.328), (3.332), (3.336), and (3.339) of [3] were multiplied by (6.5) to obtain (6.6)–(6.9).

$$\frac{E_{\phi}^{(A2)}(\phi_j^{(A2)},0)}{\left|\underline{E}^{(3+)}(\underline{J}^{\text{imp}},\underline{0})\right|_{\text{rms}}} = (S_a)\left(\frac{x_o}{a}\right)\sum_{q=0}^{\text{NMAX}-1}\sum_{\substack{p=0\\p+q\neq 0}}^{\text{MM}(q+1)-1}\sqrt{\frac{\epsilon_p\epsilon_q}{4}}\left(\frac{1}{k_{pq}b}\right)$$

$$\cdot\left\{\left(\frac{pa}{b}\right)\left(\frac{V_{pq}^{2\text{TM}}}{\sqrt{\eta}}\right)-\left(\frac{qa}{c}\right)\left(\frac{V_{pq}^{2\text{TE}}}{\sqrt{\eta}}\right)\right\}\sin\left(\frac{q\pi}{2}\right)\cos(p\phi_j^{(2)}) \quad (6.7)$$

$$\frac{E_z^{(A1)}(\pi,z_j^{(A)})}{\left|\underline{E}^{(3+)}(\underline{J}^{\text{imp}},\underline{0})\right|_{\text{rms}}} = S_a\sum_{q=0}^{\text{NMAX}-1}\sum_{\substack{p=0\\p+q\neq 0}}^{\text{MM}(q+1)-1}\sqrt{\frac{\epsilon_p\epsilon_q}{4}}\left(\frac{1}{k_{pq}b}\right)$$

$$\cdot\left\{\left(\frac{qa}{c}\right)\left(\frac{V_{pq}^{1\text{TM}}}{\sqrt{\eta}}\right)+\left(\frac{pa}{b}\right)\left(\frac{V_{pq}^{1\text{TE}}}{\sqrt{\eta}}\right)\right\}\sin\left(\frac{p\pi}{2}\right)\cos(qz_j^{(1)}) \quad (6.8)$$

$$\frac{E_z^{(A2)}(0,z_j^{(A)})}{\left|\underline{E}^{(3+)}(\underline{J}^{\text{imp}},\underline{0})\right|_{\text{rms}}} = S_a\sum_{q=0}^{\text{NMAX}-1}\sum_{\substack{p=0\\p+q\neq 0}}^{\text{MM}(q+1)-1}\sqrt{\frac{\epsilon_p\epsilon_q}{4}}\left(\frac{1}{k_{pq}b}\right)$$

$$\cdot\left\{\left(\frac{qa}{c}\right)\left(\frac{V_{pq}^{2\text{TM}}}{\sqrt{\eta}}\right)+\left(\frac{pa}{b}\right)\left(\frac{V_{pq}^{2\text{TE}}}{\sqrt{\eta}}\right)\right\}\sin\left(\frac{p\pi}{2}\right)\cos(qz_j^{(1)}) \quad (6.9)$$

where S_a is given not by eq. (3.329) of [3] but by

$$S_a = \frac{-2\pi\sqrt{\dfrac{\pi b}{c}}}{\left\{\displaystyle\sum_{r,s}\sum_{f=e,o}\frac{\left|B_{rs}^{\text{TM}f}\right|^2\beta_{rs}^{\text{TM}}}{k}+\sum_{r,s}\sum_{f=e,o}\frac{\left|B_{rs}^{\text{TE}f}\right|^2 k}{\beta_{rs}^{\text{TE}}}\right\}^{\frac{1}{2}}}. \quad (6.10)$$

The right-hand sides of (6.6)–(6.9) are those of eqs. (3.328), (3.332), (3.336), and (3.339) of [3] with S_a of eq. (3.329) of [3] replaced by S_a of (6.10) and

$$\frac{V_{pq}^{1\text{TM}}e^{j\beta_{01}^{\text{TM}}L_3}}{\eta}\rightarrow\frac{V_{pq}^{1\text{TM}}}{\sqrt{\eta}},\qquad\frac{V_{pq}^{1\text{TE}}e^{j\beta_{01}^{\text{TM}}L_3}}{\eta}\rightarrow\frac{V_{pq}^{1\text{TE}}}{\sqrt{\eta}} \quad (6.11)$$

$$\frac{V_{pq}^{2\text{TM}}e^{j\beta_{01}^{\text{TM}}L_3}}{\eta}\rightarrow\frac{V_{pq}^{2\text{TM}}}{\sqrt{\eta}},\qquad\frac{V_{pq}^{2\text{TE}}e^{j\beta_{01}^{\text{TM}}L_3}}{\eta}\rightarrow\frac{V_{pq}^{2\text{TE}}}{\sqrt{\eta}} \quad (6.12)$$

where "$\rightarrow$" means "replaced by".

Throughout this paragraph, it is assumed that $B_{01}^{\mathrm{TM}e} = 1$ and all the other B's in (6.10) are zero. With such B's, (6.10) reduces to

$$S_e = -2\pi\sqrt{\frac{\pi b}{c}}\sqrt{\frac{k}{\beta_{01}^{\mathrm{TM}}}} \,. \tag{6.13}$$

The S_e of (6.13) is that of eq. (3.329) of [3] multiplied by $-\sqrt{\beta_{01}^{\mathrm{TM}}/k}\, e^{j\beta_{01}^{\mathrm{TM}}L_a}$. Furthermore, according to (6.1), the excitation in the present report reduces to the product of $e^{j\beta_{01}^{\mathrm{TM}}L_a}/\sqrt{Z_{01}^{\mathrm{TM}eo}}$ with the excitation in [3].† Hence, in view of (3.27), the V's in (6.6)–(6.9) reduce to the products of $\sqrt{k/(\eta\beta_{01}^{\mathrm{TM}})}\, e^{j\beta_{01}^{\mathrm{TM}}L_a}$ with those in eqs. (3.328), (3.332), (3.336), and (3.339) of [3]. As a result, the numerical values of the right-hand sides of (6.6)–(6.9) reduce to those of eqs. (3.328), (3.332), (3.336), and (3.339) of [3] multiplied by $-e^{j\beta_{01}^{\mathrm{TM}}L_a}$.

†The excitation in [3] is the same as that in [1]—namely, $E_{01}^{\mathrm{TM}e+}$.

Chapter 7

The Main Program

The main program of the present report is a modification of the main program described in Sections 3.1 to 3.10 of [3] and listed in Section 3.11 of [3]. Henceforth in Chapter 7, the main program of the present report will, for the sake of brevity, be called the main program. In Sections 7.1 to 7.7, only those main program statements that are different from statements in the main program of Chapter 3 of [3] are described. These main program statements are described by defining the computer program variables in them in terms of variables in Chapters 1 to 6. In Section 7.8, the entire main program is listed. The FORTRAN statements described in Sections 7.1 to 7.7 are included in this listing.

7.1 Specification Statements

In the common block labeled PHI, PH1(150), PH2(150), PH3(150), and PH4(150) appear in the main program as opposed to PH1(100), PH2(100), PH3(100), and PH4(100) in the main program listed in Section 3.11 of [3]. Since this common block also appears in the subroutine PHI, it was necessary to replace the allocations PH1(100), PH2(100), PH3(100), and PH4(100) in the subroutine PHI by PH1(150), PH2(150), PH3(150), and PH4(150). Except for this small change, all the subprograms called by the main program are the same as those in [3]. The variable VI and the arrays CVTME, CVTEE, and CVTEO appear in the second and third COMPLEX*8 specification statements in the main program of [3] but not in the main program.

In the third REAL*4 specification statement, C1OUTS appears instead of CIOUTS, and PTOTAL is absent. It was a mistake to write CIOUTS in the third REAL*4 specification statement in the main program of [3]; C1OUTS should be there. However, this mistake is not serious because it does not affect any numerical results.

The fifth REAL*4 specification statement does not contain the variables PTMS, PRECMS, PTMR, and BKAG present in the fifth REAL*4 specification statement in the main program of [3]. It was a mistake to specify the variables PTMS, PRECMS, and PTMR as REAL*4 in [3] because these variables are not used therein.

As defined in the statement after statement 155 in the main program of [3], SA is complex. Therefore, the ensuing arrays SINP and SINQ and variables SINQQ, SINPP, BMNJPP, BMNJQQ, BMNJPQ, BMNJQP should have been specified complex instead of real in the main program of [3]. This erroneous specification will generally lead to incorrect tangential electric fields in the apertures. However, the tangential electric fields listed in Section 2.2.2 of [3] were not affected by this erroneous specification because the value of L3 was chosen such that $\mathrm{ARG} = \pi$ in the statement after statement 155 in the main program of [3]. The REAL*4 specification of the arrays SINP and SINQ and variables SINQQ, SINPP, BMNJPP, BMNJQQ, BMNJPQ, and BMNJQP is correct in the main program because the value of SA calculated by statement 155 in the main program is purely real.

The last eight specification statements in the main program are new.

7.2 The Allowable Range of ka

As for the control statements involving BKA early in DO loop 48, statement 96 of the main program is executed only if

$$\mathrm{BKA} > 1.84118378 \tag{7.1}$$

but statement 96 of the main program of [3] is executed only if

$$2.40482556 < \mathrm{BKA} < 3.05423693. \tag{7.2}$$

Because

$$x'_{11} \approx 1.84118378, \tag{7.3}$$

(7.1) states that ka must be large enough so that at least the TE_{11} modes propagate in the circular waveguide.† Here, x'_{11} is the first root of the derivative J'_1 of the Bessel function J_1; x'_{11}/a is the cutoff wavenumber of the TE_{11} circular waveguide modes. Because

$$x_{01} \approx 2.40482556 \tag{7.4}$$

$$x'_{21} \approx 3.05423693, \tag{7.5}$$

(7.2) states that ka must be such that only the TE_{11} and TM_{01} modes propagate in the circular waveguide. Here, x_{01} is the first root of J_0, and x'_{21} is the first root of J'_2; x_{01}/a and x'_{21}/a are, respectively, the cutoff wavenumbers of the TM_{01} and TE_{21} circular waveguide modes.

7.3 Calculation of the I's Defined in Section 3.4.6

The block of statements beginning with statement 46 and ending with statement 77 in the main program calculates the I's that are defined by (3.110)–(3.115). This block of statements replaces the block of statements beginning with statement 46 and ending with statement 80 in the main program of [3]. The latter block of statements calculates the excitation vector described in Chapter 4 of [2], the normalized amplitude of the wave of the $-z$-traveling even TM_{01} circular waveguide mode due to each magnetic current expansion function in the apertures, and the normalized amplitudes of the waves of the $-z$-traveling even and odd TE_{11} circular waveguide modes due to each magnetic current expansion function in the apertures.

7.3.1 Preliminary Statements

The block of 11 statements after statement 12, DO loop 114, and the three statements before DO loop 114 define variables that will be used in the block of statements beginning with statement 46 and ending with statement 77. The ninth and tenth statements after statement 12 set

$$\text{PMAX2} = 2\,(m_{\max} + 1) \tag{7.6}$$

†There are four TE_{11} modes: the $\pm z$-traveling even and odd ones.

$$T = \sqrt{\frac{8\pi b}{c}} \tag{7.7}$$

where m_{max} is the maximum value of m on the right-hand sides of (3.110)–(3.115). The three statements before DO loop 114 set

$$\text{ARG} = r\phi_o \tag{7.8}$$

$$\text{CS1} = \cos(r\phi_o) \tag{7.9}$$

$$\text{SN1} = \sin(r\phi_o)\,. \tag{7.10}$$

Here, r, which appears in (3.110)–(3.115), is related to the index R of DO loop 19 by

$$\text{R} = r + 1. \tag{7.11}$$

In DO loop 114,

$$\text{PH1(MJ)} = \phi_m^{(1)} \cos(r\phi_o) - \phi_m^{(2)} \sin(r\phi_o) \tag{7.12}$$

$$\text{PH2(MJ)} = \phi_m^{(2)} \cos(r\phi_o) + \phi_m^{(1)} \sin(r\phi_o) \tag{7.13}$$

$$\text{PH3(MJ)} = \phi_m^{(3)} \cos(r\phi_o) - \phi_m^{(4)} \sin(r\phi_o) \tag{7.14}$$

$$\text{PH4(MJ)} = \phi_m^{(4)} \cos(r\phi_o) + \phi_m^{(3)} \sin(r\phi_o) \tag{7.15}$$

where

$$\text{MJ} = \text{M} + \text{PMAX2} \tag{7.16}$$

and m is related to M by

$$\text{M} = m + 1. \tag{7.17}$$

Equations (3.90), (3.91), and (3.93)–(3.98) reduce (7.12)–(7.15) to

$$\text{PH1(MJ)} = \phi_m^{b1} \tag{7.18}$$

$$\text{PH2(MJ)} = \phi_m^{b2} \tag{7.19}$$

$$\text{PH3(MJ)} = \phi_m^{b3} \tag{7.20}$$

$$\text{PH4(MJ)} = \phi_m^{b4}\,. \tag{7.21}$$

7.3.2 Expressions (3.110) and (3.111) Associated with the TM Propagating Modes in the Circular Waveguide

The block of statements extending from the statement after statement 46 to statement 29 calculates $I_{i,rs}^{\mathrm{TM}e}$ of (3.110) and $I_{i,rs}^{\mathrm{TM}o}$ of (3.111). This block of statements will, as directed by control statement 46, be executed only if the TM_{rs} modes propagate in the circular waveguide. Here, the TM_{rs} modes are the $\pm z$-traveling even and odd TM_{rs} modes[†] for the fixed value r related to the index R of DO loop 19 by (7.11) and the fixed value s related to S by

$$S = s \tag{7.22}$$

where S is the index of DO loop 20. In the previously mentioned block of statements,

$$\mathrm{GM} = \frac{\beta_{rs}^{\mathrm{TM}}}{k} \tag{7.23}$$

$$\mathrm{TMS} = \sqrt{\left(\frac{8\pi b}{c}\right)\left(\frac{k}{\beta_{rs}^{\mathrm{TM}}}\right)\left(\frac{\epsilon_r}{2}\right)} \tag{7.24}$$

$$\mathrm{BTM(S,R)} = \frac{\beta_{rs}^{\mathrm{TM}}}{k} \tag{7.25}$$

$$\mathrm{RTM} = r + 1 \tag{7.26}$$

$$\mathrm{STM(R)} = s. \tag{7.27}$$

In (7.24),

$$\epsilon_r = \begin{cases} 1, & r = 0 \\ 2, & r \geq 1. \end{cases} \tag{7.28}$$

The value of TMS in (7.24) is its value immediately before BTM(S,R) is defined. The quantity B_{rs}^{TM}/k in (7.25) is stored in BTM(S,R) for later use in calculating the curly bracketed quantity in expression (6.10) for S_a. The variables RTM and STM(R) will be used only after exit from DO loop 19.

[†]Even TM_{rs} modes exist for $r = 0, 1, 2, \cdots$. However, odd TM_{rs} modes exist only for $r = 1, 2, \cdots$.

The values of RTM and STM(R) after exit from DO loop 19 are given as follows. If no TM mode propagates in the circular waveguide, then the values RTM = 1 and STM(1) = 0 set by the two statements after statement 12 prevail. However, if at least one TM mode propagates, then nested DO loops 19 and 20 set

$$\text{RTM} = \left\{ \begin{array}{c} \text{the maximum value of R for which} \\ \text{a } \text{TM}_{\text{R}-1,1} \text{ circular waveguide mode propagates} \end{array} \right\} \tag{7.29}$$

$$\text{STM(R)} = \left\{ \begin{array}{c} \text{the maximum value of S for which} \\ \text{a } \text{TM}_{\text{R}-1,\text{S}} \text{ circular waveguide mode propagates} \end{array} \right\}. \tag{7.30}$$

Equation (7.30) holds for $\text{R} = 1, 2, \cdots, \text{RTM}$.

Upon entry into DO loop 52,

$$\text{TMN} = \sqrt{\left(\frac{8\pi b}{c}\right)\left(\frac{k}{\beta_{rs}^{\text{TM}}}\right)\left(\frac{\epsilon_n \epsilon_r}{4}\right)\hat{G}_n^{\text{TM}}} \tag{7.31}$$

where n is related to the index N of outer DO loop 29 by

$$\text{N} = n + 1. \tag{7.32}$$

In (7.31), n appears on the right-hand sides of (3.110) and (3.111). The index M of DO loop 52 is given by (7.17) in which m appears on the right-hand sides of (3.110) and (3.111). After being increased in DO loop 52,

$$\text{MTE} = m + \left\{ \begin{array}{ll} 0, & n = 0 \\ \sum_{l=1}^{n} \text{MM}(l), & n \geq 1 \end{array} \right. \tag{7.33}$$

where m and n are related to the DO loop indices M and N by (7.17) and (7.32). With MTE given by (7.33), eq. (3.9) of [3] applies:

$$\text{BMN(MTE)} = k_{mn} b. \tag{7.34}$$

Immediately before JTME is increased inside DO loop 52, TTM is given by

$$\text{TTM} = T^{\text{TM}} \hat{G}_n^{\text{TM}} \tag{7.35}$$

where T^{TM} is given by (3.60). After being increased in DO loop 52,

$$\mathrm{JTME} = j_{\mathrm{ME}} + m + \begin{cases} 0, & n = 0 \\ \sum_{l=1}^{n} \mathrm{MM}(l), & n \geq 1 \end{cases} \tag{7.36}$$

where j_{ME} is whatever JTME was upon entry into DO loop 29. The right-hand side of (7.36) is j_{ME} plus the right-hand side of eq. (3.147) of [3] with M and N given by (7.17) and (7.32). In DO loop 52,

$$\mathrm{ITME(JTME)} = I_{i,rs}^{\mathrm{TM}e} \tag{7.37}$$

where $I_{i,rs}^{\mathrm{TM}e}$ is given by (3.110). If $\mathrm{R} \neq 1$, JTMO is, after being increased in DO loop 52, given by

$$\mathrm{JTMO} = j_{\mathrm{MO}} + m + \begin{cases} 0, & n = 0 \\ \sum_{l=1}^{n} \mathrm{MM}(l), & n \geq 1 \end{cases} \tag{7.38}$$

where j_{MO} is whatever JTMO was upon entry into DO loop 29. If $\mathrm{R} \neq 1$ in DO loop 52,

$$\mathrm{ITMO(JTMO)} = I_{i,rs}^{\mathrm{TM}o} \tag{7.39}$$

where $I_{i,rs}^{\mathrm{TM}o}$ is given by (3.111).

7.3.3 Expressions (3.112)–(3.115) Associated with the TE Propagating Modes in the Circular Waveguide

The block of statements extending from the statement after statement 65 to statement 77 calculates $I_{i,rs}^{\mathrm{TE}e\rho}$ of (3.112), $I_{i,rs}^{\mathrm{TE}ez}$ of (3.113), $I_{i,rs}^{\mathrm{TE}o\phi}$ of (3.114), and $I_{i,rs}^{\mathrm{TE}oz}$ of (3.115). This block of statements will, as directed by control statement 65, be executed only if the TE_{rs} modes propagate in the circular waveguide. Here, the TE_{rs} modes are the $\pm z$-traveling even and odd TE_{rs} modes[†] for the fixed values of r and s related to the DO loop indices R and

[†]Even TE_{rs} modes exist for $r = 0, 1, 2, \cdots$. However, odd TE_{rs} modes exist only for $r = 1, 2, \cdots$.

S by (7.11) and (7.22). In the previously mentioned block of statements,

$$\mathrm{GE} = \frac{k}{\beta_{rs}^{\mathrm{TE}}} \tag{7.40}$$

$$\mathrm{TES} = \sqrt{\left(\frac{8\pi b}{c}\right)\left(\frac{\beta_{rs}^{\mathrm{TE}}}{k(x_{rs}'^2 - r^2)}\right)\left(\frac{\epsilon_r}{2}\right)} \tag{7.41}$$

$$\mathrm{BTE(S,R)} = \frac{k}{\beta_{rs}^{\mathrm{TE}}} \tag{7.42}$$

$$\mathrm{TTEP} = r\sqrt{\left(\frac{8\pi b}{c}\right)\left(\frac{\beta_{rs}^{\mathrm{TE}}}{k(x_{rs}'^2 - r^2)}\right)\left(\frac{\epsilon_r}{2}\right)} \tag{7.43}$$

$$\mathrm{TTEZ} = \sqrt{\left(\frac{8\pi b}{c}\right)\left(\frac{\beta_{rs}^{\mathrm{TE}}}{k(x_{rs}'^2 - r^2)}\right)\left(\frac{\epsilon_r}{2}\right)\left(\frac{\sin\phi_o}{\phi_o}\right)\left(\frac{x_{rs}'^2}{\beta_{rs}^{\mathrm{TE}} a}\right)} \tag{7.44}$$

$$\mathrm{RTE} = r + 1 \tag{7.45}$$

$$\mathrm{STE(R)} = s. \tag{7.46}$$

The value of TES in (7.41) is its value immediately before BTE(S,R) is defined. The quantity $k/\beta_{rs}^{\mathrm{TE}}$ in (7.42) is stored in BTE(S,R) for later use in calculating the curly bracketed quantity in expression (6.10) for S_e. The variables RTE and STE(R) will be used only after exit from DO loop 19.

After exit from DO loop 19,

$$\mathrm{RTE} = \left\{ \begin{array}{c} \text{the maximum value of R for which} \\ \text{a } \mathrm{TE}_{\mathrm{R-1,1}} \text{ circular waveguide mode propagates} \end{array} \right\} \tag{7.47}$$

$$\mathrm{STE(R)} = \left\{ \begin{array}{c} \text{the maximum value of S for which} \\ \text{a } \mathrm{TE}_{\mathrm{R-1,S}} \text{ circular waveguide mode propagates} \end{array} \right\}. \tag{7.48}$$

Equation (7.48) is valid for $\mathrm{R} = 2, 3, \cdots, \mathrm{RTE}$. Because of (7.1), the TE_{11} circular waveguide modes propagate so that $\mathrm{RTE} \geq 2$. If the TE_{01} circular waveguide modes propagate, then STE(1) is given by (7.48). However, if the TE_{01} circular waveguide modes do not propagate, then the value $\mathrm{STE(1)} = 0$ set by the fourth statement after statement 12 prevails after exit from DO

loop 19. The third statement after statement 12 assures that RTE = 1 in the unlikely case that the TE_{11} circular waveguide modes are so close to being cutoff that, due to floating point roundoff error, ITM = 2 when the indices R and S of nested DO loops 19 and 20 are respectively equal to 2 and 1 so that these DO loops fail to define RTE.†

Upon entry into DO loop 78,

$$\text{TTEPN} = r\sqrt{\left(\frac{8\pi b}{c}\right)\left(\frac{\beta_{rs}^{\text{TE}}}{k(x_{rs}'^2 - r^2)}\right)\left(\frac{\epsilon_r \epsilon_n}{4}\right)}\,\hat{G}_n^{\text{TE}} \tag{7.49}$$

$$\text{TTEZN} = \sqrt{\left(\frac{8\pi b}{c}\right)\left(\frac{\beta_{rs}^{\text{TE}}}{k(x_{rs}'^2 - r^2)}\right)\left(\frac{\epsilon_r \epsilon_n}{4}\right)}\left(\frac{\sin\phi_o}{\phi_o}\right)\left(\frac{x_{rs}'^2}{\beta_{rs}^{\text{TE}} a}\right)\hat{G}_n^{(4)} \tag{7.50}$$

where n is related to the index N of DO loop 77 by (7.32). In (7.49) and (7.50), n appears on the right-hand sides of (3.112)–(3.115). The index M of DO loop 78 is given by (7.17) in which m appears on the right-hand sides of (3.112)–(3.115). After being increased in DO loop 78, MTE is given by (7.33). With MTE given by (7.33), BMN(MTE) is given by (7.34).

Upon arrival at statement 71,

$$\text{TEPM} = rT^{\text{TE}}\hat{G}_n^{\text{TE}} \tag{7.51}$$

$$\text{TEZM} = T^{\text{TE}}\left(\frac{\sin\phi_o}{\phi_o}\right)\left(\frac{x_{rs}'^2}{\beta_{rs}^{\text{TE}} a}\right)\hat{G}_n^{(4)} \tag{7.52}$$

where T^{TE} is given by (3.75). After statement 71 is executed,

$$\text{JTEE} = j_{zz} + m + \begin{cases} 0, & n = 0 \\ \sum_{l=1}^{n}\text{MM}(l), & n \geq 1 \end{cases} \tag{7.53}$$

†The validity of (7.1) assures that the TE_{11} circular waveguide modes propagate. As calculated in the subroutine DGN described and listed in Chapter 9 of [3], ITE=1 when R = 2 and S = 1 if the TE_{11} circular waveguide modes propagate and ITE = 2 when R = 2 and S = 1 if the TE_{11} circular waveguide modes do not propagate. Therefore, the validity of (7.1) should assure that ITE = 1 when R = 2 and S = 1. However, if (7.1) were valid but barely so, then roundoff error could result in ITE = 2 when R = 2 and S = 1.

where j_{EE} is whatever JTEE was upon entry into DO loop 77. The right-hand side of (7.53) is j_{EE} plus the right-hand side of eq. (3.147) of [3] with M and N given by (7.17) and (7.32). In DO loop 78,

$$\text{ITEEP(JTEE)} = I_{i,rs}^{\text{TEe}\phi} \tag{7.54}$$

$$\text{ITEEZ(JTEE)} = I_{i,rs}^{\text{TEe}z} \tag{7.55}$$

where $I_{i,rs}^{\text{TEe}\phi}$ and $I_{i,rs}^{\text{TEe}z}$ are given by (3.112) and (3.113). If R $\neq$ 1, JTEO is, after being increased in DO loop 78, given by

$$\text{JTEO} = j_{EO} + m + \begin{cases} 0, & n = 0 \\ \sum_{l=1}^{n} \text{MM}(l), & n \geq 1 \end{cases} \tag{7.56}$$

where j_{EO} is whatever JTEO was upon entry into DO loop 77. If R $\neq$ 1 in DO loop 78,

$$\text{ITEOP(JTEO)} = I_{i,rs}^{\text{TEo}\phi} \tag{7.57}$$

$$\text{ITEOZ(JTEO)} = I_{i,rs}^{\text{TEo}z} \tag{7.58}$$

where $I_{i,rs}^{\text{TEo}\phi}$ and $I_{i,rs}^{\text{TEo}z}$ are given by (3.114) and (3.115).

7.4 Calculation of the Normalized Elements of the Excitation Vector

The normalized elements $\{-j\sqrt{\eta}I_i^{\alpha\beta}\}$ of the excitation vector are given by (3.13).

7.4.1 The B_{rs}'s in (3.13)

The coefficients B_{rs}^{TMe}, B_{rs}^{TMo}, B_{rs}^{TEe}, and B_{rs}^{TEo} in (3.13) will be put in the computer program variables BTME(s,r+1), BTMO(s,r+1), BTEE(s,r+1), and BTEO($s,r+1$), respectively. Passing through the block of statements beginning with statement 68 and ending with statement 43,† one arrives at nested DO loops 191 and 192. These nested DO loops set to zero all the

†This block of statements is the same as in the main program of [3].

BTME's and BTMO's asociated with the TM propagating modes in the circular waveguide. Nested DO loops 193 and 194 set to zero all the BTEE's and BTEO's associated with the TE propagating modes in the circular waveguide. Actually, the BTMO(S, 1) and the BTEO(S, 1) that could be set to zero for some values of S are not coefficients in (3.13). We did not bother to avoid executing the second statement in DO loop 192 when R = 1 nor did we bother to avoid executing the second statement in DO loop 194 when R = 1.

The BTME's and BTMO's appear in (2.25). Nonzero values of them are read in inside DO loop 156. The BTEE's and BTEO's appear in (2.26). Nonzero values of them are read in inside DO loop 167.

7.4.2 The Contribution of the Even and Odd TM Propagating Modes in the Circular Waveguide

Nested DO loops 72 and 73 accumulate in X1TM, X1TE, X2TM, and X2TE the terms of the summation in (3.13) due to the even and odd TM propagating modes in the circular waveguide. After exit from nested DO loops 72 and 73,

$$\mathrm{X1TM} = \sum_{r,s} \left(B_{rs}^{\mathrm{TMe}} I_i^{\mathrm{1TM,TMe}} + B_{rs}^{\mathrm{TMo}} I_i^{\mathrm{1TM,TMo}} \right) \tag{7.59}$$

$$\mathrm{X1TE} = \sum_{r,s} \left(B_{rs}^{\mathrm{TMe}} I_i^{\mathrm{1TE,TMe}} + B_{rs}^{\mathrm{TMo}} I_i^{\mathrm{1TE,TMo}} \right) \tag{7.60}$$

$$\mathrm{X2TM} = \sum_{r,s} \left(B_{rs}^{\mathrm{TMe}} I_i^{\mathrm{2TM,TMe}} + B_{rs}^{\mathrm{TMo}} I_i^{\mathrm{2TM,TMo}} \right) \tag{7.61}$$

$$\mathrm{X2TE} = \sum_{r,s} \left(B_{rs}^{\mathrm{TMe}} I_i^{\mathrm{2TE,TMe}} + B_{rs}^{\mathrm{TMo}} I_i^{\mathrm{2TE,TMo}} \right) \tag{7.62}$$

where the subscript i on the I_i's denotes dependence on the integers m and n in (3.14). The eight I_i's on the right-hand sides of (7.59)–(7.62) are given by (3.57), (3.58), (3.63), (3.64), (3.65), (3.66), (3.69), and (3.70) where m and n are related to the indices M and N of outer DO loops 74 and 16 by (7.17) and (7.32). The quantities $I_{i,rs}^{\mathrm{TMe}}$ and $I_{i,rs}^{\mathrm{TMo}}$ in (3.57) and (3.65) were previously calculated. See (7.37) and (7.39). Expressions (7.59)–(7.62) are the contributions of all the even and odd TM circular waveguide propagating modes to the summation on the right-hand side of (3.13) with $\alpha\beta$ = 1TM,

1TE, 2TM, and 2TE, respectively. The calculation of these expressions is described in the next three paragraphs.

Both MTM and MTE are set to zero before entry into DO loop 16. The integer MTM will be treated in Section 7.4.4. After being increased inside DO loop 74, MTE is given by (7.33). The indices R and S of DO loops 72 and 73 are related to the summation indices r and s in (7.59)–(7.62) by (7.11) and (7.22). Because JTME and JTMO are increased by KTE of eq. (4.13) of [3] immediately after use in DO loop 73,

$$\text{ITME(JTME)} = I_{i,rs}^{\text{TM}e} \tag{7.63}$$

in the first statement in DO loop 73 and

$$\text{ITMO(JTMO)} = I_{i,rs}^{\text{TM}o} \tag{7.64}$$

in the tenth statement in DO loop 73. Consequently, these statements set

$$\text{Z1} = B_{rs}^{\text{TM}e} I_{i,rs}^{\text{TM}e} \tag{7.65}$$

and

$$\text{Z1} = B_{rs}^{\text{TM}o} I_{i,rs}^{\text{TM}o}\,, \tag{7.66}$$

respectively.

The third and fourth statements in DO loop 73 set

$$\text{W1} = (-1)^{r+1} n \tag{7.67}$$

$$\text{W2} = (-1)^{r+1} \left(\frac{mc}{b}\right) \tag{7.68}$$

so that, in the fifth through eighth statements in DO loop 73,

$$-\text{W1}*\text{Z1} = B_{rs}^{\text{TM}e} I_i^{\text{1TM,TM}e} \tag{7.69}$$

$$-\text{W2}*\text{Z1} = B_{rs}^{\text{TM}e} I_i^{\text{1TE,TM}e} \tag{7.70}$$

$$\text{FN1}*\text{Z1} = B_{rs}^{\text{TM}e} I_i^{\text{2TM,TM}e} \tag{7.71}$$

$$\text{FM1}*\text{Z1} = B_{rs}^{\text{TM}e} I_i^{\text{2TE,TM}e} \tag{7.72}$$

where the I_i's are given by (3.57), (3.58), (3.63), and (3.64).

The last six statements in DO loop 73 are not executed for R = 1 because there are no odd TM_{0s} modes in the circular waveguide. In the last four statements in DO loop 73,

$$\mathrm{W1*Z1} = B_{rs}^{\mathrm{TMo}} I_i^{1\mathrm{TM,TMo}} \tag{7.73}$$

$$\mathrm{W2*Z1} = B_{rs}^{\mathrm{TMo}} I_i^{1\mathrm{TE,TMo}} \tag{7.74}$$

$$\mathrm{FN1*Z1} = B_{rs}^{\mathrm{TMo}} I_i^{2\mathrm{TM,TMo}} \tag{7.75}$$

$$\mathrm{FM1*Z1} = B_{rs}^{\mathrm{TMo}} I_i^{2\mathrm{TE,TMo}} \tag{7.76}$$

where the I_i's are given by (3.65), (3.66), (3.69), and (3.70). It is evident from (7.69)–(7.76) that X1TM, X1TE, X2TM, and X2TE will be given by (7.59)–(7.62) upon exit from nested DO loops 72 and 73.

7.4.3 The Contribution of the Even and Odd TE Propagating Modes in the Circular Waveguide

Nested DO loops 168 and 169 add to what was already stored in X1TM, X1TE, X2TM, and X2TE the terms of the summation in (3.13) due to the even and odd TE propagating modes in the circular waveguide. After exit from nested DO loops 168 and 169,

$$\begin{aligned}\mathrm{X1TM} = &\sum_{r,s}\left(B_{rs}^{\mathrm{TMe}} I_i^{1\mathrm{TM,TMe}} + B_{rs}^{\mathrm{TMo}} I_i^{1\mathrm{TM,TMo}}\right)\\ &+\sum_{r,s}\left(B_{rs}^{\mathrm{TEe}} I_i^{1\mathrm{TM,TEe}} + B_{rs}^{\mathrm{TEo}} I_i^{1\mathrm{TM,TEo}}\right)\end{aligned} \tag{7.77}$$

$$\begin{aligned}\mathrm{X1TE} = &\sum_{r,s}\left(B_{rs}^{\mathrm{TMe}} I_i^{1\mathrm{TE,TMe}} + B_{rs}^{\mathrm{TMo}} I_i^{1\mathrm{TE,TMo}}\right)\\ &+\sum_{r,s}\left(B_{rs}^{\mathrm{TEe}} I_i^{1\mathrm{TE,TEe}} + B_{rs}^{\mathrm{TEo}} I_i^{1\mathrm{TE,TEo}}\right)\end{aligned} \tag{7.78}$$

$$\begin{aligned}\mathrm{X2TM} = &\sum_{r,s}\left(B_{rs}^{\mathrm{TMe}} I_i^{2\mathrm{TM,TMe}} + B_{rs}^{\mathrm{TMo}} I_i^{2\mathrm{TM,TMo}}\right)\\ &+\sum_{r,s}\left(B_{rs}^{\mathrm{TEe}} I_i^{2\mathrm{TM,TEe}} + B_{rs}^{\mathrm{TEo}} I_i^{2\mathrm{TM,TEo}}\right)\end{aligned} \tag{7.79}$$

$$\mathrm{X2TE} = \sum_{r,s} \left(B_{rs}^{\mathrm{TM}e} I_i^{2\mathrm{TE},\mathrm{TM}e} + B_{rs}^{\mathrm{TM}o} I_i^{2\mathrm{TE},\mathrm{TM}o} \right)$$
$$+ \sum_{r,s} \left(B_{rs}^{\mathrm{TE}e} I_i^{2\mathrm{TE},\mathrm{TE}e} + B_{rs}^{\mathrm{TE}o} I_i^{2\mathrm{TE},\mathrm{TE}o} \right) \tag{7.80}$$

where the subscript i on the I_i's denotes dependence on the integers m and n in (3.14). There are two summations on the right-hand side of each of (7.77)–(7.80). The first summation is what was already stored (see (7.59)–(7.62)). The calculation of the second summation is discussed in this section. The eight I_i's in the second summations on the right-hand sides of (7.77)–(7.80) are given by (3.71), (3.72), (3.79), (3.80), (3.81), (3.82), (3.86), and (3.87) where m and n are related to the indices M and N of outer DO loops 74 and 16 by (7.17) and (7.32). The quantities $I_{i,rs}^{\mathrm{TE}e\phi}$, $I_{i,rs}^{\mathrm{TE}ez}$, $I_{i,rs}^{\mathrm{TE}o\phi}$, and $I_{i,rs}^{\mathrm{TE}oz}$ in (3.71) and (3.81) were previously computed. See (7.54), (7.55), (7.57), and (7.58). The second summations in (7.77)–(7.80) are the contributions of all the even and odd TE circular waveguide propagating modes to the summation on the right-hand side of (3.13) with $\alpha\beta$ = 1TM, 1TE, 2TM, and 2TE, respectively. The calculation of these second summations is described in the next two paragraphs.

The indices R and S of DO loops 168 and 169 are related to the summation indices r and s in (7.77)–(7.80) by (7.11) and (7.22). Because JTEE is increased by KTE of eq. (4.13) of [3] immediately after use in DO loop 169, the first and second statements in DO loop 169 set

$$\mathrm{W5} = I_{i,rs}^{\mathrm{TE}e\phi} \tag{7.81}$$
$$\mathrm{W6} = I_{i,rs}^{\mathrm{TE}ez}\,. \tag{7.82}$$

The fourth through eighth statements in DO loop 169 set

$$\mathrm{W3} = I_i^{2\mathrm{TM},\mathrm{TE}e} \tag{7.83}$$
$$\mathrm{W4} = I_i^{2\mathrm{TE},\mathrm{TE}e} \tag{7.84}$$
$$\mathrm{W1} = I_i^{1\mathrm{TM},\mathrm{TE}e} \tag{7.85}$$
$$\mathrm{W2} = I_i^{1\mathrm{TE},\mathrm{TE}e} \tag{7.86}$$
$$\mathrm{Z1} = B_{rs}^{\mathrm{TE}e} \tag{7.87}$$

where the I_i's are given by (3.71), (3.72), (3.79), and (3.80). In the ninth through twelfth statements in DO loop 169,

$$\mathrm{W1*Z1} = B_{rs}^{\mathrm{TE}e} I_i^{1\mathrm{TM},\mathrm{TE}e} \tag{7.88}$$

$$\mathrm{W2*Z1} = B_{rs}^{\mathrm{TE}e} I_i^{1\mathrm{TE},\mathrm{TE}e} \tag{7.89}$$

$$\mathrm{W3*Z1} = B_{rs}^{\mathrm{TE}e} I_i^{2\mathrm{TM},\mathrm{TE}e} \tag{7.90}$$

$$\mathrm{W4*Z1} = B_{rs}^{\mathrm{TE}e} I_i^{2\mathrm{TE},\mathrm{TE}e}\,. \tag{7.91}$$

The last twelve statements in DO loop 169 are not executed if R = 1 because there are no odd TE_{0s} modes in the circular waveguide. Because the third of these statements increases JTEO by KTE of eq. (4.13) of [3], the first two of these statements set

$$\mathrm{W5} = I_{i,rs}^{\mathrm{TE}o\phi} \tag{7.92}$$

$$\mathrm{W6} = I_{i,rs}^{\mathrm{TE}oz}\,. \tag{7.93}$$

The fourth through eighth of these statements set

$$\mathrm{W3} = -I_i^{2\mathrm{TM},\mathrm{TE}o} \tag{7.94}$$

$$\mathrm{W4} = -I_i^{2\mathrm{TE},\mathrm{TE}o} \tag{7.95}$$

$$\mathrm{W1} = I_i^{1\mathrm{TM},\mathrm{TE}o} \tag{7.96}$$

$$\mathrm{W2} = I_i^{1\mathrm{TE},\mathrm{TE}o} \tag{7.97}$$

$$\mathrm{Z1} = B_{rs}^{\mathrm{TE}o} \tag{7.98}$$

where the I_i's are given by (3.81), (3.82), (3.86), and (3.87). In the last four statements in DO loop 169,

$$\mathrm{W1*Z1} = B_{rs}^{\mathrm{TE}o} I_i^{1\mathrm{TM},\mathrm{TE}o} \tag{7.99}$$

$$\mathrm{W2*Z1} = B_{rs}^{\mathrm{TE}o} I_i^{1\mathrm{TE},\mathrm{TE}o} \tag{7.100}$$

$$-\mathrm{W3*Z1} = B_{rs}^{\mathrm{TE}o} I_i^{2\mathrm{TM},\mathrm{TE}o} \tag{7.101}$$

$$-\mathrm{W4*Z1} = B_{rs}^{\mathrm{TE}o} I_i^{2\mathrm{TE},\mathrm{TE}o}\,. \tag{7.102}$$

It is evident from (7.59)–(7.62), (7.88)–(7.91), and (7.99)–(7.102) that X1TM, X1TE, X2TM, and X2TE will be given by (7.77)–(7.80) upon exit from nested DO loops 168 and 169.

7.4.4 Storage of the Normalized Elements of the Excitation Vector

The eighth from the last through the fifth from the last statements in DO loop 74 are not executed if either N = 1 or M = 1 because there is neither a TM_{0n} nor a TM_{m0} rectangular waveguide mode expansion function. The seventh from the last and the fifth from the last statements in DO loop 74 use X1TM of (7.77), X2TM of (7.79), and

$$U = j \tag{7.103}$$

to set

$$\mathrm{TI(MTM)} = -j\sqrt{\eta}I_i^{1\mathrm{TM}} \tag{7.104}$$

$$\mathrm{TI(J1)} = -j\sqrt{\eta}I_i^{2\mathrm{TM}}\,. \tag{7.105}$$

The right-hand sides of (7.104) and (7.105) are given by (3.13) with $\alpha\beta =$ 1TM and 2TM, respectively. These are the normalized elements of the excitation vector corresponding to the TM rectangular waveguide mode expansion functions. In (7.104), MTM is given in terms of the indices N and M of DO loops 16 and 74 by eq. (3.146) of [3]. In (7.105),

$$\mathrm{J1} = \mathrm{MTM} + \mathrm{K1} \tag{7.106}$$

where K1 is the total number of TE and TM rectangular waveguide mode expansion functions given by eq. (3.10) of [3] where KTE and KTM are given by eqs. (4.13) and (4.14) of [3].

The third from the last and the last statements in DO loop 74 use X1TE of (7.78), X2TE of (7.80), and (7.103) to set

$$\mathrm{TI(J1)} = -j\sqrt{\eta}I_i^{1\mathrm{TE}} \tag{7.107}$$

$$\mathrm{TI(J2)} = -j\sqrt{\eta}I_i^{2\mathrm{TE}}\,. \tag{7.108}$$

The right-hand sides of (7.107) and (7.108) are given by (3.13) with $\alpha\beta =$ 1TE and 2TE, respectively. These are the normalized elements of the excitation vector corresponding to the TE rectangular waveguide mode expansion functions. In (7.107),

$$\mathrm{J1} = \mathrm{MTE} + \mathrm{KTM} \tag{7.109}$$

where MTE is given in terms of n and m by (7.33). Alternatively, MTE is given in terms of the indices N and M of DO loops 16 and 74 by eq. (3.147) of [3]. In (7.108),

$$\mathrm{J2} = \mathrm{MTE} + \mathrm{KTM} + \mathrm{K1}. \tag{7.110}$$

7.5 The Normalized Coefficients of the TE_{10} Modes in the Rectangular Waveguides

The normalized coefficients of the waves of the TE_{10} modes in the rectangular waveguides are given by (4.3)–(4.6) as opposed to those of [3], which are given by eqs. (5.32), (5.33), (5.36), and (5.37) of [2].† Equations (4.3) and (4.4) differ from eqs. (5.32) and (5.33) of [2] only in that the right-hand sides of (4.3) and (4.4) have the multiplier $\sqrt{\beta_{10}/k}\,(V_{10}^{1\mathrm{TE}}/\sqrt{\eta})$ while the right-hand sides of (5.32) and (5.33) of [2] have the multiplier $\sqrt{\beta_{10}/\beta_{01}^{\mathrm{TM}}}\,(V_{10}^{1\mathrm{TE}}e^{j\beta_{01}^{\mathrm{TM}}L_3}/\eta)$. Equations (4.5) and (4.6) differ from eqs. (5.36) and (5.37) of [2] only in that the right-hand sides of (4.5) and (4.6) have the multiplier $\sqrt{\beta_{10}/k}\,(V_{10}^{2\mathrm{TE}}/\sqrt{\eta})$ while the right-hand sides of eqs. (5.36) and (5.37) of [2] have the multiplier $\sqrt{\beta_{10}/\beta_{01}^{\mathrm{TM}}}\,(V_{10}^{2\mathrm{TE}}e^{j\beta_{01}^{\mathrm{TM}}L_3}/\eta)$.

In the main program of [3], the right-hand sides of eqs. (5.32), (5.33), (5.36), and (5.37) were calculated with $V_{10}^{1\mathrm{TE}}e^{j\beta_{01}^{\mathrm{TM}}L_3}/\eta$ and $V_{10}^{2\mathrm{TE}}e^{j\beta_{01}^{\mathrm{TM}}L_3}/\eta$ stored in the array V. In the main program (of the present report), the matrix equation (2.22) of [1], concisely written as

$$Y\vec{V} = \vec{I} \tag{7.111}$$

is recast as

$$\mathrm{YV} = \mathrm{TI} \tag{7.112}$$

where Y, V, and TI are arrays in the main program. The array Y contains the elements of $-j\eta Y$. The array TI contains the elements of $-j\sqrt{\eta}\vec{I}$. From

†There is a misprint in eq. (5.37) of [2]. The left-hand side of (5.37) of [2] should be $C_{10}^{2\mathrm{TE}-}$.

(7.111), (7.112), and the definitions of Y and TI in (7.112), one sees that the array V contains the elements of $\vec{V}/\sqrt{\eta}$. Two of these elements are $V_{10}^{1TE}/\sqrt{\eta}$ and $V_{10}^{2TE}/\sqrt{\eta}$. Hence, $V_{10}^{1TE}/\sqrt{\eta}$ and $V_{10}^{2TE}/\sqrt{\eta}$ in (4.3)–(4.6) are treated in the main program exactly the same as $V_{10}^{1TE}e^{j\beta_{01}^{TM}L_3}/\eta$ and $V_{10}^{2TE}e^{j\beta_{01}^{TM}L_3}/\eta$ in eqs. (5.32), (5.33), (5.36), and (5.37) of [2] were treated in the main program of [3].

The block of statements beginning with the statement after statement 16 and ending with statement 70 writes TI, solves the matrix equation (7.112), writes the solution V, calculates the normalized coefficients (4.3)–(4.6) of the waves of the TE_{10} modes in the rectangular waveguides, and writes these normalized coefficients. In this block of statements, only the statement that defines S1 is different from that in the main program of [3]. In the main program,

$$S1 = \tfrac{1}{2}\sqrt{\beta_{10}/k} \tag{7.113}$$

as opposed to

$$S1 = \tfrac{1}{2}\sqrt{\beta_{10}/\beta_{01}^{TM}} \tag{7.114}$$

in the main program of [3]. The S1 of (7.113) is the product of 1/2 with the factor $\sqrt{\beta_{10}/k}$ in (4.3)–(4.6) while the S1 of (7.114) is the product of 1/2 with the factor $\sqrt{\beta_{10}/\beta_{01}^{TM}}$ in eqs. (5.32), (5.33), (5.36), and (5.37) of [2]. In the main program. C1OUT, C1IN, C2OUT, and C2IN are C_{10}^{1TE-} of (4.3), C_{10}^{1TE+} of (4.4), C_{10}^{2TE+} of (4.5), and C_{10}^{2TE-} of (4.6), respectively. These are the normalized coefficients of the waves of the TE_{10} modes in the rectangular waveguides.

The seven statements following statement 70 are the same as the seven statements following statement 81 in the main program of [3]. These seven statements calculate and write the squares of the magnitudes of the normalized coefficients of the waves of the TE_{10} modes in the rectangular waveguides and the time-average power $P_t^{(1)}+P_t^{(2)}$ transmitted into the rectangular waveguides. Here, $P_t^{(1)}$ and $P_t^{(2)}$ are given in terms of these squares by (4.7) and (4.8), respectively. The fifth statement after statement 70 sets

$$PT = P_t^{(1)} + P_t^{(2)}. \tag{7.115}$$

7.6 The Normalized Coefficients of the Propagating Modes in the Circular Waveguide

The block of statements beginning with the statement after statement 82 and ending with statement 190 calculates and writes $C_{rs}^{\mathrm{TM}e}$ of (5.21), $C_{rs}^{\mathrm{TM}o}$ of (5.22), $C_{rs}^{\mathrm{TE}e}$ of (5.31), $C_{rs}^{\mathrm{TE}o}$ of (5.32), and the time-average powers P_{rm} of (5.34), P_{inc} of (5.44), and P_r of (5.45). This block of statements also calculates the computer program variable W7 defined by

$$\mathrm{W7} = \sum_{r,s} \sum_{f=e,o} \frac{\left|B_{rs}^{\mathrm{TM}f}\right|^2 \beta_{rs}^{\mathrm{TM}}}{k} + \sum_{r,s} \sum_{f=e,o} \frac{\left|B_{rs}^{\mathrm{TE}f}\right|^2 k}{\beta_{rs}^{\mathrm{TE}}}. \qquad (7.116)$$

The above expression appears in (6.10). Now, $C_{rs}^{\mathrm{TM}e}$, $C_{rs}^{\mathrm{TM}o}$, $C_{rs}^{\mathrm{TE}e}$, and $C_{rs}^{\mathrm{TE}o}$ are the normalized coefficients of the waves of the $-z$-traveling even TM_{rs}, odd TM_{rs}, even TE_{rs}, and odd TE_{rs} circular waveguide modes, respectively. The quantities P_{rm}, P_{inc}, and P_r depend on these normalized coefficients. Here, P_{rm} is the time-average power that the combination of magnetic currents $-\underline{M}^{(1)}$ and $-\underline{M}^{(2)}$ in Fig. 2 would radiate in the absence of $\underline{J}^{\mathrm{imp}}$, P_{inc} is the time-average power of the z-traveling waves in the circular waveguide, and P_r is the time-average power of the $-z$-traveling waves in the circular waveguide. The right-hand side of (7.116) is the term in brackets in the denominator of (6.10). In this term, the first double summation is over all the even and odd TM_{rs} propagating modes in the circular waveguide and the second double summation is over all the even and odd TE_{rs} propagating modes in the circular waveguide.

7.6.1 The Normalized Coefficients of the TM Propagating Modes in the Circular Waveguide

The normalized coefficients $C_{rs}^{\mathrm{TM}e}$ and $C_{rs}^{\mathrm{TM}o}$ of the waves of the even and odd TM_{rs} propagating modes in the circular waveguide are calculated in nested DO loops 172 and 173 whose indices R and S obtain r and s according to (7.11) and (7.22). These DO loops also add to W7 the TM_{rs}^{e} and TM_{rs}^{o} terms in (7.116). Specifically, the fifth statement in DO loop 173 adds $(|B_{rs}^{\mathrm{TM}e}|^2 +$

$(|B_{rs}^{\mathrm{TMo}}|^2)\beta_{rs}^{\mathrm{TM}}/k$ to W7. Inside nested DO loops 172 and 173, nested DO loops 174 and 176 accumulate the triple summations of (5.21) and (5.22) in XTME and XTMO, respectively. After exit from nested DO loops 174 and 176,

$$\mathrm{XTME} = \sum_{\gamma=1}^{2}\sum_{q=0}\sum_{\substack{p=0\\p+q\neq0}}\left\{I_j^{\gamma\mathrm{TM,TMe}}\left(\frac{V_{pq}^{\gamma\mathrm{TM}}}{\sqrt{\eta}}\right)+I_j^{\gamma\mathrm{TE,TMe}}\left(\frac{V_{pq}^{\gamma\mathrm{TE}}}{\sqrt{\eta}}\right)\right\} \quad (7.117)$$

$$\mathrm{XTMO} = \sum_{\gamma=1}^{2}\sum_{q=0}\sum_{\substack{p=0\\p+q\neq0}}\left\{I_j^{\gamma\mathrm{TM,TMo}}\left(\frac{V_{pq}^{\gamma\mathrm{TM}}}{\sqrt{\eta}}\right)+I_j^{\gamma\mathrm{TE,TMo}}\left(\frac{V_{pq}^{\gamma\mathrm{TE}}}{\sqrt{\eta}}\right)\right\}. \quad (7.118)$$

The calculation of the right-hand sides of (7.117) and (7.118) is described in the next six paragraphs.

Inside nested DO loops 174 and 176, the pq^{th} terms in (7.117) and (7.118) are added to XTME and XTMO, respectively, where q and p are related to the DO loop indices N and M by

$$\mathrm{N} = q+1 \quad (7.119)$$

$$\mathrm{M} = p+1\,. \quad (7.120)$$

There are possibly four pq^{th} terms in (7.117). They are

$$I_j^{1\mathrm{TM.TMe}}\left(\frac{V_{pq}^{1\mathrm{TM}}}{\sqrt{\eta}}\right),\ I_j^{1\mathrm{TE.TMe}}\left(\frac{V_{pq}^{1\mathrm{TE}}}{\sqrt{\eta}}\right),$$

$$I_j^{2\mathrm{TM.TMe}}\left(\frac{V_{pq}^{2\mathrm{TM}}}{\sqrt{\eta}}\right),\ \text{and}\ I_j^{2\mathrm{TE.TMe}}\left(\frac{V_{pq}^{2\mathrm{TE}}}{\sqrt{\eta}}\right). \quad (7.121)$$

Preliminary to the calculation of these terms, the third and fifth statements in DO loop 176 set

$$\mathrm{FM1} = \frac{pc}{b} \quad (7.122)$$

$$\mathrm{W5} = I_{j,rs}^{\mathrm{TMe}} \quad (7.123)$$

where $I_{j,rs}^{\mathrm{TMe}}$ is $I_{i,rs}^{\mathrm{TMe}}$ of (3.110) with (i,m,n) replaced by (j,p,q). The quantity $I_{i,rs}^{\mathrm{TMe}}$ was previously calculated and, according to (7.37), stored in ITME (JTME).

The seventh through thirteenth statements in DO loop 176 account for the first and third terms in (7.121). These statements are not executed if either $p = 0$ or $q = 0$ because these terms do not exist then. Quantities $V_{0q}^{1\mathrm{TM}}$, $V_{p0}^{1\mathrm{TM}}$, $V_{0q}^{2\mathrm{TM}}$, and $V_{p0}^{2\mathrm{TM}}$ do not exist because there are neither any TM_{0q} nor any TM_{p0} rectangular waveguide modes. If executed, the ninth through 12th statements in DO loop 176 set

$$\mathrm{Z1} = \frac{V_{pq}^{1\mathrm{TM}}}{\sqrt{\eta}} \tag{7.124}$$

$$\mathrm{Z3} = \frac{V_{pq}^{2\mathrm{TM}}}{\sqrt{\eta}} \tag{7.125}$$

$$\mathrm{W3} = I_j^{2\mathrm{TM},\mathrm{TM}e} \tag{7.126}$$

$$\mathrm{W1} = -I_j^{1\mathrm{TM},\mathrm{TM}e} \tag{7.127}$$

where $I_j^{1\mathrm{TM},\mathrm{TM}e}$ and $I_j^{2\mathrm{TM},\mathrm{TM}e}$ are obtained by replacing (i, m, n) by (j, p, q) in (3.57) and (3.63), respectively. In the 13th statement in DO loop 176,

$$-\mathrm{W1}*\mathrm{Z1} = I_j^{1\mathrm{TM},\mathrm{TM}e}\left(\frac{V_{pq}^{1\mathrm{TM}}}{\sqrt{\eta}}\right) \tag{7.128}$$

$$\mathrm{W3}*\mathrm{Z3} = I_j^{2\mathrm{TM},\mathrm{TM}e}\left(\frac{V_{pq}^{2\mathrm{TM}}}{\sqrt{\eta}}\right). \tag{7.129}$$

Thus, the 13th statement in DO loop 176 adds to XTME the first and third terms in (7.121).

Always executed, the 14th through 21st statements in DO loop 176 account for the second and fourth terms in (7.121). The 17th through 20th statements in DO loop 176 set

$$\mathrm{Z2} = \frac{V_{pq}^{1\mathrm{TE}}}{\sqrt{\eta}} \tag{7.130}$$

$$\mathrm{Z4} = \frac{V_{pq}^{2\mathrm{TE}}}{\sqrt{\eta}} \tag{7.131}$$

$$\mathrm{W4} = I_j^{2\mathrm{TE},\mathrm{TM}e} \tag{7.132}$$

$$\mathrm{W2} = -I_j^{1\mathrm{TE},\mathrm{TM}e} \tag{7.133}$$

where $I_j^{1\text{TE},\text{TM}e}$ and $I_j^{2\text{TE},\text{TM}e}$ are obtained by replacing (i, m, n) by (j, p, q) in (3.58) and (3.64), respectively. In the 21st statement in DO loop 176,

$$-\text{W2}*\text{Z2} = I_j^{1\text{TE},\text{TM}e}\left(\frac{V_{pq}^{1\text{TE}}}{\sqrt{\eta}}\right) \tag{7.134}$$

$$\text{W4}*\text{Z4} = I_j^{2\text{TE},\text{TM}e}\left(\frac{V_{pq}^{2\text{TE}}}{\sqrt{\eta}}\right). \tag{7.135}$$

Thus, the 21st statement in DO loop 176 adds to XTME the second and fourth terms in (7.121).

There are possibly four pq^{th} terms in (7.118). They are

$$I_j^{1\text{TM},\text{TM}o}\left(\frac{V_{pq}^{1\text{TM}}}{\sqrt{\eta}}\right),\ I_j^{1\text{TE},\text{TM}o}\left(\frac{V_{pq}^{1\text{TE}}}{\sqrt{\eta}}\right),$$

$$I_j^{2\text{TM},\text{TM}o}\left(\frac{V_{pq}^{2\text{TM}}}{\sqrt{\eta}}\right),\ \text{and}\ I_j^{2\text{TE},\text{TM}o}\left(\frac{V_{pq}^{2\text{TE}}}{\sqrt{\eta}}\right). \tag{7.136}$$

The last nine statements in DO loop 176 account for these terms. These statements are not executed when $r = 0$ because then XTMO is not needed because there is no $C_{0s}^{\text{TM}o}$. Preliminary to the calculation of the terms in (7.136), the eighth from the last statement in DO loop 176 sets

$$\text{W5} = I_{j,rs}^{\text{TM}o} \tag{7.137}$$

where $I_{j,rs}^{\text{TM}o}$ is $I_{i,rs}^{\text{TM}o}$ of (3.111) with (i, m, n) replaced by (j, p, q). The quantity $I_{i,rs}^{\text{TM}o}$ was previously calculated and, according to (7.39), stored in ITMO (JTMO).

The sixth from the last through fourth from the last statements in DO loop 176 account for the first and third terms in (7.136). These statements are not executed if $p = 0$ or $q = 0$ because these terms do not exist then. Quantities $V_{0q}^{1\text{TM}}$, $V_{p0}^{1\text{TM}}$, $V_{0q}^{2\text{TM}}$, and $V_{p0}^{2\text{TM}}$ do not exist because there are neither any TM_{0q} nor any TM_{p0} rectangular waveguide modes. If executed, the sixth from the last and fifth from the last statements in DO loop 176 set

$$\text{W3} = I_j^{2\text{TM},\text{TM}o} \tag{7.138}$$

$$\text{W1} = I_j^{1\text{TM},\text{TM}o} \tag{7.139}$$

where $I_j^{1\mathrm{TM,TMo}}$ and $I_j^{2\mathrm{TM,TMo}}$ are obtained by replacing (i,m,n) by (j,p,q) in (3.65) and (3.69), respectively. In the fourth from the last statement in DO loop 176,

$$\mathrm{W1*Z1} = I_j^{1\mathrm{TM,TMo}} \left(\frac{V_{pq}^{1\mathrm{TM}}}{\sqrt{\eta}} \right) \tag{7.140}$$

$$\mathrm{W3*Z3} = I_j^{2\mathrm{TM,TMo}} \left(\frac{V_{pq}^{2\mathrm{TM}}}{\sqrt{\eta}} \right). \tag{7.141}$$

Thus, the fourth from the last statement in DO loop 176 adds to XTMO the first and third terms in (7.136).

The last three statements in DO loop 176 account for the second and fourth terms in (7.136). The third from the last and second from the last statements in DO loop 176 set

$$\mathrm{W4} = I_j^{2\mathrm{TE,TMo}} \tag{7.142}$$

$$\mathrm{W2} = I_j^{1\mathrm{TE,TMo}} \tag{7.143}$$

where $I_j^{1\mathrm{TE,TMo}}$ and $I_j^{2\mathrm{TE,TMo}}$ are obtained by replacing (i,m,n) by (j,p,q) in (3.66) and (3.70), respectively. In the last statement in DO loop 176,

$$\mathrm{W2*Z2} = I_j^{1\mathrm{TE,TMo}} \left(\frac{V_{pq}^{1\mathrm{TE}}}{\sqrt{\eta}} \right) \tag{7.144}$$

$$\mathrm{W4*Z4} = I_j^{2\mathrm{TE,TMo}} \left(\frac{V_{pq}^{2\mathrm{TE}}}{\sqrt{\eta}} \right). \tag{7.145}$$

Thus, the last statement in DO loop 176 adds to XTMO the second and fourth terms in (7.136). It is evident from the discussion in this paragraph and the preceding five paragraphs that XTME and XTMO are given by (7.117) and (7.118).

The first nine statements after statement 174 use XTME to calculate C_{rs}^{TMe} of (5.21). These statements also add $|B_{rs}^{\mathrm{TMe}}|^2$ of (5.44) to PIN, $|C_{rs}^{\mathrm{TMe}} + B_{rs}^{\mathrm{TMe}}|^2$ of (5.34) to PRM, and $|C_{rs}^{\mathrm{TMe}}|^2$ of (5.45) to PR. The quantities P_{inc} of (5.44), P_{rm} of (5.34), and P_r of (5.45) will be accumulated in PIN, PRM, and PR, respectively. The first two statements after statement 174 set

$$\mathrm{XTME} = C_{rs}^{\mathrm{TMe}} + B_{rs}^{\mathrm{TMe}} \tag{7.146}$$

$$\mathrm{BT} = B_{rs}^{\mathrm{TMe}} \tag{7.147}$$

where C_{rs}^{TMe} is given by (5.21). The third statement after statement 174 sets

$$\mathrm{PINS} = \left|B_{rs}^{\mathrm{TMe}}\right|^2 \tag{7.148}$$

so that the next statement adds $|B_{rs}^{\mathrm{TMe}}|^2$ to PIN. The fifth through seventh statements after statement 174 set

$$\mathrm{CTME} = C_{rs}^{\mathrm{TMe}} \tag{7.149}$$

$$\mathrm{XTMES} = \left|C_{rs}^{\mathrm{TMe}} + B_{rs}^{\mathrm{TMe}}\right|^2 \tag{7.150}$$

$$\mathrm{CTMES} = \left|C_{rs}^{\mathrm{TMe}}\right|^2 \tag{7.151}$$

so that the next two statements add $\left|C_{rs}^{\mathrm{TMe}} + B_{rs}^{\mathrm{TMe}}\right|^2$ to PRM and $\left|C_{rs}^{\mathrm{TMe}}\right|^2$ to PR.

The second through 10$^{\text{th}}$ statements after statement 162 use XTMO to calculate C_{rs}^{TMo} of (5.22). These statements also add $|B_{rs}^{\mathrm{TMo}}|^2$ of (5.44) to PIN, $|C_{rs}^{\mathrm{TMo}} + B_{rs}^{\mathrm{TMo}}|^2$ of (5.34) to PRM, and $|C_{rs}^{\mathrm{TMo}}|^2$ of (5.45) to PR. The above-mentioned statements are not executed if $r=0$ because there is neither C_{0s}^{TMo} nor B_{0s}^{TMo}. The second and third statements after statement 162 set

$$\mathrm{XTMO} = C_{rs}^{\mathrm{TMo}} + B_{rs}^{\mathrm{TMo}} \tag{7.152}$$

$$\mathrm{BT} = B_{rs}^{\mathrm{TMo}} \tag{7.153}$$

where C_{rs}^{TMo} is given by (5.22). The fourth statement after statement 162 sets

$$\mathrm{PINS} = \left|B_{rs}^{\mathrm{TMo}}\right|^2 \tag{7.154}$$

so that the next statement adds $|B_{rs}^{\mathrm{TMo}}|^2$ to PIN. The sixth through eighth statements after statement 162 set

$$\mathrm{CTMO} = C_{rs}^{\mathrm{TMo}} \tag{7.155}$$

$$\mathrm{XTMOS} = \left|C_{rs}^{\mathrm{TMo}} + B_{rs}^{\mathrm{TMo}}\right|^2 \tag{7.156}$$

$$\mathrm{CTMOS} = \left|C_{rs}^{\mathrm{TMo}}\right|^2 \tag{7.157}$$

so that the next two statements add $\left|C_{rs}^{\mathrm{TMo}} + B_{rs}^{\mathrm{TMo}}\right|^2$ to PRM and $\left|C_{rs}^{\mathrm{TMo}}\right|^2$ to PR.

7.6.2 The Normalized Coefficients of the TE Propagating Modes in the Circular Waveguide

The normalized coefficients $C_{rs}^{\mathrm{TE}e}$ and $C_{rs}^{\mathrm{TE}o}$ of the waves of the even and odd TE_{rs} propagating modes in the circular waveguide are calculated in nested DO loops 180 and 181 whose indices R and S obtain r and s according to (7.11) and (7.22). These DO loops also add to W7 the TE_{rs}^{e} and TE_{rs}^{o} terms in (7.116). Specifically, the fifth statement in DO loop 181 adds $(|B_{rs}^{\mathrm{TE}e}|^2 + (|B_{rs}^{\mathrm{TE}o}|^2)k/\beta_{rs}^{\mathrm{TE}}$ to W7. Inside nested DO loops 172 and 173, nested DO loops Inside nested DO loops 180 and 181, nested DO loops 182 and 75 accumulate the triple summations of (5.31) and (5.32) in XTEE and XTEO, respectively. After exit from nested DO loops 182 and 75,

$$\mathrm{XTEE} = \sum_{\gamma=1}^{2}\sum_{q=0}\sum_{\substack{p=0\\p+q\neq 0}} \left\{ I_j^{\gamma\mathrm{TM},\mathrm{TE}e}\left(\frac{V_{pq}^{\gamma\mathrm{TM}}}{\sqrt{\eta}}\right) + I_j^{\gamma\mathrm{TE},\mathrm{TE}e}\left(\frac{V_{pq}^{\gamma\mathrm{TE}}}{\sqrt{\eta}}\right)\right\} \tag{7.158}$$

$$\mathrm{XTEO} = \sum_{\gamma=1}^{2}\sum_{q=0}\sum_{\substack{p=0\\p+q\neq 0}} \left\{ I_j^{\gamma\mathrm{TM},\mathrm{TE}o}\left(\frac{V_{pq}^{\gamma\mathrm{TM}}}{\sqrt{\eta}}\right) + I_j^{\gamma\mathrm{TE},\mathrm{TE}o}\left(\frac{V_{pq}^{\gamma\mathrm{TE}}}{\sqrt{\eta}}\right)\right\}. \tag{7.159}$$

The calculation of the right-hand sides of (7.158) and (7.159) is described in the next six paragraphs.

Inside nested DO loops 182 and 75, the pq^{th} terms in (7.158) and (7.159) are added to XTEE and XTEO, respectively, where q and p are related to the DO loop indices N and M by (7.119) and (7.120). There are possibly four pq^{th} terms in (7.158). They are

$$I_j^{1\mathrm{TM},\mathrm{TE}e}\left(\frac{V_{pq}^{1\mathrm{TM}}}{\sqrt{\eta}}\right),\ I_j^{1\mathrm{TE},\mathrm{TE}e}\left(\frac{V_{pq}^{1\mathrm{TE}}}{\sqrt{\eta}}\right),$$
$$I_j^{2\mathrm{TM},\mathrm{TE}e}\left(\frac{V_{pq}^{2\mathrm{TM}}}{\sqrt{\eta}}\right),\ \text{and } I_j^{2\mathrm{TE},\mathrm{TE}e}\left(\frac{V_{pq}^{2\mathrm{TE}}}{\sqrt{\eta}}\right). \tag{7.160}$$

Preliminary to the calculation of these terms, the third statement in DO loop 75 sets FM1 equal to the right-hand side of (7.122). The fifth and sixth statements in DO loop 75 set

$$\mathrm{W5} = I_{j,rs}^{\mathrm{TE}e\phi} \tag{7.161}$$
$$\mathrm{W6} = I_{j,rs}^{\mathrm{TE}ez} \tag{7.162}$$

where $I_{j,rs}^{\mathrm{TE}e\phi}$ is $I_{i,rs}^{\mathrm{TE}e\phi}$ of (3.112) with (i,m,n) replaced by (j,p,q) and $I_{j,rs}^{\mathrm{TE}ez}$ is $I_{i,rs}^{\mathrm{TE}ez}$ of (3.113) with (i,m,n) replaced by (j,p,q). The quantities $I_{i,rs}^{\mathrm{TE}e\phi}$ and $I_{i,rs}^{\mathrm{TE}ez}$ were previously calculated and, according to (7.54) and (7.55), stored in ITEEP(JTEE) and ITEEZ(JTEE), respectively.

The eighth through 14$^{\mathrm{th}}$ statements in DO loop 75 account for the first and third terms in (7.160). These statements are not executed if $p=0$ or $q=0$ because these terms do not exist then. If executed, the 10$^{\mathrm{th}}$ through 13$^{\mathrm{th}}$ statements in DO loop 75 set

$$\mathrm{Z1} = \frac{V_{pq}^{1\mathrm{TM}}}{\sqrt{\eta}} \tag{7.163}$$

$$\mathrm{Z3} = \frac{V_{pq}^{2\mathrm{TM}}}{\sqrt{\eta}} \tag{7.164}$$

$$\mathrm{W3} = I_j^{2\mathrm{TM},\mathrm{TE}e} \tag{7.165}$$

$$\mathrm{W1} = I_j^{1\mathrm{TM},\mathrm{TE}e} \tag{7.166}$$

where $I_j^{1\mathrm{TM},\mathrm{TE}e}$ and $I_j^{2\mathrm{TM},\mathrm{TE}e}$ are obtained by replacing (i,m,n) by (j,p,q) in (3.71) and (3.79), respectively. In the 14$^{\mathrm{th}}$ statement in DO loop 75,

$$\mathrm{W1*Z1} = I_j^{1\mathrm{TM},\mathrm{TE}e}\left(\frac{V_{pq}^{1\mathrm{TM}}}{\sqrt{\eta}}\right) \tag{7.167}$$

$$\mathrm{W3*Z3} = I_j^{2\mathrm{TM},\mathrm{TE}e}\left(\frac{V_{pq}^{2\mathrm{TM}}}{\sqrt{\eta}}\right) \tag{7.168}$$

so that this statement adds to XTEE the first and third terms in (7.160).

Always executed, statement 175 and the seven statements after it account for the second and fourth terms in (7.160). The third through sixth statements after statement 175 set

$$\mathrm{Z2} = \frac{V_{pq}^{1\mathrm{TE}}}{\sqrt{\eta}} \tag{7.169}$$

$$\mathrm{Z4} = \frac{V_{pq}^{2\mathrm{TE}}}{\sqrt{\eta}} \tag{7.170}$$

$$W4 = I_j^{2TE,TEe} \tag{7.171}$$

$$W2 = I_j^{1TE,TEe} \tag{7.172}$$

where $I_j^{1TE,TEe}$ and $I_j^{2TE,TEe}$ are obtained by replacing (i,m,n) by (j,p,q) in (3.72) and (3.80), respectively. In the seventh statement after statement 175,

$$W2{*}Z2 = I_j^{1TE,TEe}\left(\frac{V_{pq}^{1TE}}{\sqrt{\eta}}\right) \tag{7.173}$$

$$W4{*}Z4 = I_j^{2TE,TEe}\left(\frac{V_{pq}^{2TE}}{\sqrt{\eta}}\right). \tag{7.174}$$

Thus, the seventh statement after statement 175 adds to XTEE the second and fourth terms in (7.160).

There are possibly four pq^{th} terms in (7.159). They are

$$I_j^{1TM,TEo}\left(\frac{V_{pq}^{1TM}}{\sqrt{\eta}}\right),\ I_j^{1TE,TEo}\left(\frac{V_{pq}^{1TE}}{\sqrt{\eta}}\right),$$
$$I_j^{2TM,TEo}\left(\frac{V_{pq}^{2TM}}{\sqrt{\eta}}\right),\ \text{and}\ I_j^{2TE,TEo}\left(\frac{V_{pq}^{2TE}}{\sqrt{\eta}}\right). \tag{7.175}$$

The last 10 statements in DO loop 75 account for these terms. These statements are not executed when $r=0$ because then XTEO is not needed because there is no C_{0s}^{TEo}. Preliminary to the calculation of the terms in (7.175), the ninth and eighth from the last statements in DO loop 75 set

$$W5 = I_{j,rs}^{TEo\phi} \tag{7.176}$$

$$W6 = I_{j,rs}^{TEoz} \tag{7.177}$$

where $I_{j,rs}^{TEo\phi}$ is $I_{i,rs}^{TEo\phi}$ of (3.114) with (i,m,n) replaced by (j,p,q) and $I_{j,rs}^{TEoz}$ is $I_{i,rs}^{TEoz}$ of (3.115) with (i,m,n) replaced by (j,p,q). The quantities $I_{i,rs}^{TEo\phi}$ and $I_{i,rs}^{TEoz}$ were previously calculated and, according to (7.57) and (7.58), stored in ITEOP(JTEO) and ITEOZ(JTEO), respectively.

The sixth from the last through fourth from the last statements in DO loop 75 account for the first and third terms in (7.175). These statements are not executed if $p=0$ or $q=0$ because these terms do not exist then. If

executed, the sixth from the last and fifth from the last statements in DO loop 75 set

$$\mathrm{W3} = -I_j^{2\mathrm{TM},\mathrm{TE}o} \tag{7.178}$$

$$\mathrm{W1} = I_j^{1\mathrm{TM},\mathrm{TE}o} \tag{7.179}$$

where $I_j^{1\mathrm{TM},\mathrm{TE}o}$ and $I_j^{2\mathrm{TM},\mathrm{TE}o}$ are obtained by replacing (i,m,n) by (j,p,q) in (3.81) and (3.86), respectively. In the fourth from the last statement in DO loop 75,

$$\mathrm{W1}*\mathrm{Z1} = I_j^{1\mathrm{TM},\mathrm{TE}o}\left(\frac{V_{pq}^{1\mathrm{TM}}}{\sqrt{\eta}}\right) \tag{7.180}$$

$$-\mathrm{W3}*\mathrm{Z3} = I_j^{2\mathrm{TM},\mathrm{TE}o}\left(\frac{V_{pq}^{2\mathrm{TM}}}{\sqrt{\eta}}\right) \tag{7.181}$$

so that this statement adds to XTEO the first and third terms in (7.175).

The last three statements in DO loop 75 account for the second and fourth terms in (7.175). The third from the last and second from the last statements in DO loop 75 set

$$\mathrm{W4} = -I_j^{2\mathrm{TE},\mathrm{TE}o} \tag{7.182}$$

$$\mathrm{W2} = I_j^{1\mathrm{TE},\mathrm{TE}o} \tag{7.183}$$

where $I_j^{1\mathrm{TE},\mathrm{TE}o}$ and $I_j^{2\mathrm{TE},\mathrm{TE}o}$ are obtained by replacing (i,m,n) by (j,p,q) in (3.82) and (3.87), respectively. In the last statement in DO loop 75,

$$\mathrm{W2}*\mathrm{Z2} = I_j^{1\mathrm{TE},\mathrm{TE}o}\left(\frac{V_{pq}^{1\mathrm{TE}}}{\sqrt{\eta}}\right) \tag{7.184}$$

$$-\mathrm{W4}*\mathrm{Z4} = I_j^{2\mathrm{TE},\mathrm{TE}o}\left(\frac{V_{pq}^{2\mathrm{TE}}}{\sqrt{\eta}}\right) \tag{7.185}$$

so that this statement adds to XTEO the second and fourth terms in (7.175). It is evident from the discussion in this paragraph and the preceding five paragraphs that XTEE and XTEO are given by (7.158) and (7.159).

The first nine statements after statement 182 use XTEE to calculate $C_{rs}^{\mathrm{TE}e}$ of (5.31). These statements also add $|B_{rs}^{\mathrm{TE}e}|^2$ of (5.44) to PIN, $|C_{rs}^{\mathrm{TE}e}+B_{rs}^{\mathrm{TE}e}|^2$

of (5.34) to PRM, and $|C_{rs}^{TEe}|^2$ of (5.45) to PR. The quantities P_{inc} of (5.44), P_{rm} of (5.34), and P_r of (5.45) are being accumulated in PIN, PRM, and PR, respectively. The first two statements after statement 182 set

$$\text{XTEE} = C_{rs}^{TEe} + B_{rs}^{TEe} \tag{7.186}$$

$$\text{BT} = B_{rs}^{TEe} \tag{7.187}$$

where C_{rs}^{TEe} is given by (5.31). The third statement after statement 182 sets

$$\text{PINS} = \left|B_{rs}^{TEe}\right|^2 \tag{7.188}$$

so that the next statement adds $|B_{rs}^{TEe}|^2$ to PIN. The fifth through seventh statements after statement 182 set

$$\text{CTEE} = C_{rs}^{TEe} \tag{7.189}$$

$$\text{XTEES} = \left|C_{rs}^{TEe} + B_{rs}^{TEe}\right|^2 \tag{7.190}$$

$$\text{CTEES} = \left|C_{rs}^{TEe}\right|^2 \tag{7.191}$$

so that the next two statements add $\left|C_{rs}^{TEe} + B_{rs}^{TEe}\right|^2$ to PRM and $\left|C_{rs}^{TEe}\right|^2$ to PR.

The second through 10[th] statements after statement 165 use XTEO to calculate C_{rs}^{TEo} of (5.32). These statements also add $|B_{rs}^{TEo}|^2$ of (5.44) to PIN, $|C_{rs}^{TEo} + B_{rs}^{TEo}|^2$ of (5.34) to PRM, and $|C_{rs}^{TEo}|^2$ of (5.45) to PR. The above-mentioned statements are not executed if $r = 0$ because there is neither C_{0s}^{TEo} nor B_{0s}^{TEo}. The second and third statements after statement 165 set

$$\text{XTEO} = C_{rs}^{TEo} + B_{rs}^{TEo} \tag{7.192}$$

$$\text{BT} = B_{rs}^{TEo} \tag{7.193}$$

where C_{rs}^{TEo} is given by (5.32). The fourth statement after statement 165 sets

$$\text{PINS} = \left|B_{rs}^{TEo}\right|^2 \tag{7.194}$$

so that the next statement adds $|B_{rs}^{TEo}|^2$ to PIN. The sixth through eighth statements after statement 165 set

$$\text{CTEO} = C_{rs}^{TEo} \tag{7.195}$$

$$\text{XTEOS} = \left|C_{rs}^{TEo} + B_{rs}^{TEo}\right|^2 \tag{7.196}$$

$$\text{CTEOS} = \left|C_{rs}^{TEo}\right|^2 \tag{7.197}$$

so that the next two statements add $\left|C_{rs}^{\mathrm{TE}o} + B_{rs}^{\mathrm{TE}o}\right|^2$ to PRM and $\left|C_{rs}^{\mathrm{TE}o}\right|^2$ to PR.

After exit from nested DO loops 180 and 181,

$$\mathrm{PIN} = P_{\mathrm{inc}} \tag{7.198}$$

$$\mathrm{PRM} = P_{rm} \tag{7.199}$$

$$\mathrm{PR} = P_r \tag{7.200}$$

where P_{inc}, P_{rm}, and P_r are given by (5.44), (5.34), and (5.45), respectively.

7.7 All Statements Beyond Statement 190 in the Main Program

The group of statements beginning with the statement after statement 190 and ending with the last statement in the main program are, except for minor differences noted in the following paragraphs, the same as in the main program of [3].

The second statement after statement 190 is new. This statement sets

$$\mathrm{PINC(KA)} = P_{\mathrm{inc}}\,. \tag{7.201}$$

No value of BKAG is calculated after statement 91 because BKAG no longer appears in the two statements that define PTA and PRMA.† These two statements, immediately following statement 91, set

$$\mathrm{PTA} = P_{ta} \tag{7.202}$$

$$\mathrm{PRMA} = P_{rma} \tag{7.203}$$

where P_{ta} and P_{rma} are, in contrast to eqs. (3.316) and (3.323) of [3], given by

$$P_{ta} = -\mathrm{Im}\left\{\sum_{i=1}^{K2}(-j\eta Y_{ii}^{\mathrm{rec}})\left|\frac{V_i}{\sqrt{\eta}}\right|^2\right\} \tag{7.204}$$

$$P_{rma} = -P_{ta} - \mathrm{Im}\left\{\sum_{i=1}^{K2}(-j\sqrt{\eta}\,I_i)\left(\frac{V_i^*}{\sqrt{\eta}}\right)\right\}. \tag{7.205}$$

†The left-hand side of eq. (3.317) of [3] is incorrect. This left-hand side should be BKAG instead of BKAB.

In (7.204), Y_{ii}^{rec} is the same as $[Y^1 + Y^2]_{ii}$ in eq. (3.316) of [3]:

$$Y_{ii}^{rec} = \left[Y^1 + Y^2\right]_{ii} . \tag{7.206}$$

To gain physical interpretations of the quantities P_{ts} and P_{rms}, we first recast (7.204) as

$$P_{ts} = P_{ts}^{(1)} + P_{ts}^{(2)} \tag{7.207}$$

where $P_{ts}^{(1)}$ and $P_{ts}^{(2)}$ are given by (4.12) and (4.17), respectively. As stated in Section 4.3, $P_{ts}^{(1)}$ is an alternative expression for the time-average power radiated by the magnetic current $\underline{M}^{(1)}$ in the left-hand rectangular waveguide and $P_{ts}^{(2)}$ is an alternative expression for the time-average power radiated by the magnetic current $\underline{M}^{(2)}$ in the right-hand rectangular waveguide. Hence, P_{ts} is an alternative expression for the time-average power radiated by the magnetic currents in the rectangular waveguides. Using (7.207), we see that the right-hand side of (7.205) is the same as that of (5.43). Hence, P_{rms} of (7.05) is an alternative expression for the time-average power radiated by the magnetic currents in the circular waveguide.

Upon exit from DO loop 101, we want to have

$$\text{E3A1P(J)} = \frac{E_\phi^{(A1)}(\phi_J^{(A1)}, 0)}{\left|\underline{E}^{(3+)}(\underline{J}^{imp}, \underline{0})\right|_{rms}} \tag{7.208}$$

$$\text{E3A2P(J)} = \frac{E_\phi^{(A2)}(\phi_J^{(A2)}, 0)}{\left|\underline{E}^{(3+)}(\underline{J}^{imp}, \underline{0})\right|_{rms}} \tag{7.209}$$

$$\text{E3A1Z(J)} = \frac{E_z^{(A1)}(\pi, z_J^{(A)})}{\left|\underline{E}^{(3+)}(\underline{J}^{imp}, \underline{0})\right|_{rms}} \tag{7.210}$$

$$\text{E3A2Z(J)} = \frac{E_z^{(A2)}(0, z_J^{(A)})}{\left|\underline{E}^{(3+)}(\underline{J}^{imp}, \underline{0})\right|_{rms}} \tag{7.211}$$

where the right-hand sides of (7.208)–(7.211) are given by (6.6)–(6.9), respectively. According to eqs. (3.369)–(3.372) of [3], the main program of [3]

sets†

$$\text{E3A1P(J)} = \frac{-E_\phi^{(A1)}(\phi_j^{(A1)}, 0)}{\left|E_{01}^{\text{TMe}+}\right|_{\text{rms}}} \tag{7.212}$$

$$\text{E3A2P(J)} = \frac{-E_\phi^{(A2)}(\phi_j^{(A2)}, 0)}{\left|E_{01}^{\text{TMe}+}\right|_{\text{rms}}} \tag{7.213}$$

$$\text{E3A1Z(J)} = \frac{-E_z^{(A1)}(\pi, z_j^{(A)})}{\left|E_{01}^{\text{TMe}+}\right|_{\text{rms}}} \tag{7.214}$$

$$\text{E3A2Z(J)} = \frac{-E_z^{(A2)}(0, z_j^{(A)})}{\left|E_{01}^{\text{TMe}+}\right|_{\text{rms}}} \tag{7.215}$$

where the right-hand sides of (7.212)–(7.215) are given by eqs. (3.328), (3.332), (3.336), and (3.339) of [3], respectively.

The main program treats each parenthesized quantity proportional to one of the V's in (6.6)–(6.9) exactly the same as the main program of [3] treats the corresponding parenthesized quantity in eqs. (3.328), (3.332), (3.336), and (3.339) of [3]. For example, the main program stores $V_{pq}^{1\text{TM}}/\sqrt{\eta}$ of (6.6) and (6.8) in the same variable as the main program of [3] stores $V_{pq}^{1\text{TM}} e^{j\beta_{01}^{\text{TM}} L_0}/\eta$ of eqs. (3.328) and (3.336) of [3]. Hence, as far as both main programs are concerned, the right-hand sides of (6.6)–(6.9) are those of eqs. (3.328), (3.332), (3.336), and (3.339) of [3] with S_e of eq. (3.329) of [3] replaced by S_e of (6.10). The difference in S_e is accounted for in statement 155. Statement 155 in the main program of [3] and the statement after it have been replaced

---

†The right-hand sides of eqs. (3.369)–(3.372) of [3] are incorrect. The correct right-hand sides are the negatives of those written in these equations. Equations (7.212)–(7.215) are the corrected eqs. (3.369), (3.371), (3.370), and (3.372) of [3], respectively. The sign errors in eqs. (3.369)–(3.372) of [3] were due to incorrect signs on the left-hand sides of eqs. (7.12)–(7.14) of [2]. In fact, every $E_\phi^{(A1)}(\phi_j^{(A1)}, 0)$, $E_\phi^{(A2)}(\phi_j^{(A2)}, 0)$, $E_z^{(A1)}(\pi, z_j^{(A)})$, and $E_z^{(A2)}(0, z_j^{(A)})$ in Section 3.10 of [3] has the wrong sign attached to it. However, nested DO loops 101 and 104 of the main program of [3] still accumulate the right-hand sides of eqs. (3.328), (3.332), (3.336), and (3.339) of [3] in E3A1P(J), E3A2P(J), E3A1Z(J), and E3A2Z(J), respectively.

by statement 155. The latter statement 155 sets

$$\mathrm{SA} = S_a \tag{7.216}$$

where S_a is given by (6.10) instead of eq. (3.329) of [3]. In this statement, W7 is given by (7.116).

In the main program of [3], the value of the time-average power incident in the circular waveguide is not written out because the time-average transmitted and reflected powers calculated therein are those per unit time-average incident power. In the main program, however, the value of the time-average incident power is written out because the time-average transmitted and reflected powers calculated therein are absolute time-average powers rather than ratios of them to the time-average incident power. The time-average incident power in the main program depends on the amplitudes of the waves of the propagating modes incident in the circular waveguide. The statement after statement 149 writes out (PINC(I), I = 1, KAM) where

$$\mathrm{PINC(I)} = P_{inc} \tag{7.217}$$

where P_{inc} is the time-average incident power given by (5.44) when the index KA of DO loop 48 is I. When KA = I, ka is given by the right-hand side of (2.15) with KA replaced by I.

7.8 Listing of the Main Program

```
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON /MODES/PC,BKM2,KTM,KTE,MM(50),BMN(100),BMN2(100)
      COMMON /PI/PI
      COMMON /NMAX/NMAX
      COMMON /BES/XN,SMAX
      COMMON /PHI/BX,BX5,PMAX,R,SGR,PH1(150),PH2(150),PH3(150),
     1PH4(150)
      COMMON /DGN/S,[illegible],L3,C,C5,PI5,D3(50),D4(50),PGC
      COMPLEX*16 ZL1,ZL2,U,BKU,SA,YTE,YTM,DTM(50),DTMN,DTE(50),DTEN
      COMPLEX*16 D3,D3N,Z1,Z2,Z3,Z4,Z5,S1,S3,S4,S5
      COMPLEX*8 E3A1P(100),E3A1Z(100),E3A2P(100),E3A2Z(100)
      COMPLEX*8 YMM,YEM,YME,YEE,Y(24336),TI(156),V(156)
      COMPLEX*8 CTME,CTEE,CTEO,C1OUT,C1IN,C2OUT
      COMPLEX*8 C2IN,YREC(156)
      REAL*8 L1,L2,L3,XJ(200),XJP(200),GTM(50),GTE(50)
```

```
      REAL*8 TMP(50),TMM(50),TEP(50),TEM(50),DQTM(50),DQTE(50)
      REAL*4 PHI1(100),PHI2(100),Z(100)
      REAL*4 PTRAN(100),PREFL(100),BKAPLT(100)
      REAL*4 C1OUTS,C1INS,C2OUTS,C2INS,PT,CTMES,CTEES,CTEOS,PR
      REAL*4 SINP(100),SINQ(100),E3A1PS(100),E3A2PS(100),E3A1ZS(100)
      REAL*4 E3A2ZS(100),SINQQ,SINPP,BMNJP
      REAL*4 BMNJQ,BMNJPP,BMNJQQ,BMNJPQ,BMNJQP
      INTEGER R,R1,S,SMAX,P,PMAX,P1,P2,P3,Q,Q1,PTM,PTE,QTM,QTE,IPS(156)
      INTEGER QPN,KE3(101)
      INTEGER RTM,RTE,PMAX2,STM(10),STE(10)
      INTEGER RTMR,RTER,STMR(10),STER(10)
      REAL*4 ITME(2000),ITMO(2000),ITEEP(2000),ITEEZ(2000)
      REAL*4 ITEOP(2000),ITEOZ(2000),PIN,PTA,PRMA,PINC(100)
      REAL*4 BTM(10,10),BTE(10,10)
      COMPLEX*8 BTME(10,10),BTMO(10,10),BTEE(10,10),BTEO(10,10)
      COMPLEX*8 BT,XTME,XTMO,XTEE,XTEO,CTMO
      COMPLEX*8 X1TM,X1TE,X2TM,X2TE
      OPEN(UNIT=20,FILE='IN.DAT',STATUS='OLD')
      OPEN(UNIT=21,FILE='OUT.DAT',STATUS='OLD')
      READ(20,10) B,C,L1,L2,L3,BKM,XM,ZL1,ZL2
   10 FORMAT(4D14.7)
      WRITE(21,11)
   11 FORMAT('B,C,L1,L2,L3,BKM,XM,ZL1,ZL2')
      WRITE(21,10) B,C,L1,L2,L3,BKM,XM,ZL1,ZL2
      READ(20,144) KAN,BKAO,DBKA,KE3M,NPHI,NZ
  144 FORMAT(I4,2D14.7,3I4)
      WRITE(21,145) KAN,BKAO,DBKA,KE3M,NPHI,NZ
  145 FORMAT('KAN=',I4,', BKAO=',D14.7,', DBKA=',D14.7/
     1'KE3M=',I4,', NPHI=',I4,', NZ=',I4)
      READ(20,146)(KE3(I),I=1,KE3M)
  146 FORMAT(15I4)
      WRITE(21,147)(KE3(I),I=1,KE3M)
  147 FORMAT('KE3'/(15I4))
      PI=3.14159265358979D+0
      BC=B/C
      PC=PI*BC
      BKM2=BKM*BKM
      CALL MODES
      WRITE(21,102) NMAX
  102 FORMAT('NMAX=',I4)
      WRITE(21,108)(NM(I),I=1,NMAX)
  108 FORMAT('NM'/(15I4))
      K1=KTM+KTE
      WRITE(21,153) KTM,KTE,K1
```

```
153 FORMAT('KTM=',I4,', KTE=',I4,', K1=',I4)
    IF(NMAX.GT.0) GO TO 115
    WRITE(21,116)
116 FORMAT('BKN IS TOO SMALL')
    STOP
115 CALL BESIN
    PI2=PI*2.D+0
    PIBC=PI*BC
    PI5=PI*.5D+0
    KAE=1
    DO 48 KA=1,KAN
    BKA=BKA0+(KA-1)*DBKA
    BKA2=BKA*BKA
    IF(BKA.GT.1.84118378D+0) GO TO 96
    WRITE(21,95)
 95 FORMAT('BKA IS TOO SMALL')
    STOP
 96 IF(C.LT.B) GO TO 98
    WRITE(21,99)
 99 FORMAT('C IS NOT LESS THAN B')
    STOP
 98 BKB=BKA*B
    WRITE(21,150) BKB
150 FORMAT('BKB=',E14.7)
    IF(BKB.GT.PI) GO TO 110
    WRITE(21,111)
111 FORMAT('BKB IS TOO SMALL')
    STOP
110 IF(BKB.LT.PI2.AND.BKB.LT.PIBC) GO TO 112
    WRITE(21,113)
113 FORMAT('BKB IS TOO LARGE')
    STOP
112 BKB2=BKB*BKB
    BKR=1.D+0/BKB
    U=(0.D+0,1.D+0)
    BKU=-BKR*U
    B5=B*.5D+0
    BX5=DASIN(B5)
    BX=2.D+0*BX5
    XB=1.D+0/BX
    X1=L1/B-XB
    X2=L2/B-XB
    JTE=0
    JTM=0
```

```
      DO 13 Q=1,NMAX
      P2=1
      IF(Q.EQ.1) P2=2
      P3=MM(Q)
      DO 14 P=P2,P3
      JTE=JTE+1
      JTE1=JTE+KTM
      JTE2=JTE1+K1
      GAM2=BKC2(JTE)-BKB2
      IF(P.NE.2.OR.Q.NE.1) GO TO 15
      BET=DSQRT(-GAM2)
      A1=BET*X1
      CA=DCOS(A1)
      SA=DSIN(A1)*U
      S1=BET*BKU
      YREC(JTE1)=(CA+ZL1*SA)/(SA+ZL1*CA)*S1
      A2=BET*X2
      CA=DCOS(A2)
      SA=DSIN(A2)*U
      YREC(JTE2)=(CA+ZL2*SA)/(SA+ZL2*CA)*S1
      GO TO 17
   15 GAM=DSQRT(GAM2)
      YTE=-GAM*BKR
      YREC(JTE1)=YTE
      YREC(JTE2)=YTE
   17 IF(P.EQ.1.OR.Q.EQ.1) GO TO 14
      YTM=BKB/GAM
      JTM=JTM+1
      JTM2=JTM+K1
      YREC(JTM)=YTM
      YREC(JTM2)=YTM
   14 CONTINUE
   13 CONTINUE
      K2=K1*2
      WRITE(21,100)(YREC(J),J=1,K2)
  100 FORMAT('YREC'/(4E14.7))
      C5=C*.5D+0
      ZSS=1.D+0/C5
      PMAX=MM(1)
      XZ=B/BX
      TZTM=8.*BX5*DSQRT(BC/PI)
      TZTE=TZTM/BC
      TA=PI2/BKA
      SQ2=1.D+0/DSQRT(2.D+0)
```

```
      TC1=DSQRT(PI*BC)
      TC5=TC1/SQ2
      TC1=TC1*BKA
      SN1=BS
      CS1=DSQRT(1.D+0-BS*BS)
      K3=K2*K1
      DO 12 IY=1,K3
      Y(IY)=0.
   12 CONTINUE
      KTM=1
      STM(1)=0
      KTE=1
      STE(1)=0
      JTME=0
      JTMO=0
      JTEE=0
      JTEO=0
      PMAX2=PMAX*2
      T=DSQRT(8.D+0*PIBC)
      SGR=-1.D+0
      DO 19 R=1,500
      SGR=-SGR
      R1=R-1
      RS=R1*R1
      CALL BES(R,XJ,XJP)
      IF(SMAX.GT.200) STOP 60
      IF(SMAX.EQ.0) GO TO 25
      CALL PHI
      ARG=R1*BX5
      CS1=DCOS(ARG)
      SN1=DSIN(ARG)
      DO 114 N=1,PMAX
      NJ=N+PMAX2
      PH1(NJ)=PH1(N)*CS1-PH2(N)*SN1
      PH2(NJ)=PH2(N)*CS1+PH1(N)*SN1
      PH3(NJ)=PH3(N)*CS1-PH4(N)*SN1
      PH4(NJ)=PH4(N)*CS1+PH3(N)*SN1
  114 CONTINUE
      DO 20 S=1,SMAX
      CALL DGN(1,XJ,XXTM,ITM,GAMTM,TMP,TMM,DTM,GTM,DQTM,GCSTM,GC2TM,
     1ZEETM,ZZTM,ZOETM,ZOOTM)
      CALL DGN(2,XJP,XXTE,ITE,GAMTE,TEP,TEM,DTE,GTE,DQTE,GCSTE,GC2TE,
     1ZEETE,ZZTE,ZOETE,ZOOTE)
      XR=XXTE-RS
```

```
      W2=-RS/XR*GAMTE
      W6=XZ*XXTE
      W5=W6/XR
      W3=R1*W5
      W5=W5*W6
      W6=ZSS/XXTE*W5
      W1=BKA2/GAMTM
      W5=W5/GAMTE
      IF(R1.NE.0) GO TO 46
      W1=W1*.5D+0
      W5=W5*.5D+0
      W6=W6*.5D+0
46    IF(ITM.EQ.2) GO TO 65
      GM=GAMTM/BKA
      TMS=T/DSQRT(GM)
      IF(R1.EQ.0) TMS=TMS*SQ2
      BTM(S,R)=GM
      RTM=R
      STM(R)=S
      MTE=0
      DO 29 N=1,NMAX
      M2=1
      N1=N-1
      TMN=TMS*GTM(N)
      IF(N1.NE.0) GO TO 76
      M2=2
      TMN=TMN*SQ2
76    M3=MM(N)
      DO 52 M=M2,M3
      MTE=MTE+1
      M1=M-1
      TTM=TMN/BMN(MTE)
      IF(M1.EQ.0) TTM=TTM*SQ2
      JTME=JTME+1
      MJ=M+PMAX2
      ITME(JTME)=TTM*PE2(MJ)
      IF(R1.EQ.0) GO TO 52
      JTMO=JTMO+1
      ITMO(JTMO)=TTM*PE1(MJ)
52    CONTINUE
29    CONTINUE
65    IF(ITE.EQ.2) GO TO 68
      GE=BKA/GAMTE
      TES=T/DSQRT(GE*XR)
```

```
      IF(R1.EQ.0) TES=TES*SQ2
      ETE(S,R)=GE
      TTEP=TES*R1
      TTEZ=TES*XZ*XXTE/GAMTE
      RTE=R
      STE(R)=S
      NTE=0
      DO 77 N=1,NMAX
      M2=1
      N1=N-1
      TTEPN=TTEP*GTE(N)
      TTEZN=TTEZ*G4(N)
      IF(N1.NE.0) GO TO 67
      M2=2
      TTEPN=TTEPN*SQ2
      TTEZN=TTEZN*SQ2
   67 M3=MM(N)
      DO 78 M=M2,M3
      NTE=NTE+1
      BNMN=BMN(NTE)
      TEPN=TTEPN/BNMN
      TEZN=TTEZN/BNMN
      M1=M-1
      IF(M1.NE.0) GO TO 71
      TEPN=TEPN*SQ2
      TEZN=TEZN*SQ2
   71 JTEE=JTEE+1
      MJ=M+PMAX2
      XTEEP(JTEE)=TEPN*PE1(MJ)
      XTEEZ(JTEE)=TEZN*PE4(MJ)
      IF(R1.EQ.0) GO TO 78
      JTEO=JTEO+1
      XTEOP(JTEO)=TEPN*PE2(MJ)
      XTEOZ(JTEO)=TEZN*PE3(MJ)
   78 CONTINUE
   77 CONTINUE
   68 NTM=0
      NTE=0
      DO 21 N=1,NMAX
      M2=1
      N1=N-1
      IF(N1.EQ.0) M2=2
      M3=MM(N)
      QTM=0
```

```
      QTE=0
      FN1B=N1*BC
      NEO=1+N1-2*(N1/2)
      GO TO (18,51), ITM
   18 TMMN=TMM(N)
      TMPN=TMP(N)
      DTMN=DTM(N)
      GO TO 83
   51 DQTMN=DQTM(N)
      TM1=GCSTM*DQTMN
      TM2=GC2TM*DQTMN
   83 GO TO (84,85), ITE
   84 TEMN=TEM(N)
      TEPN=TEP(N)
      DTEN=DTE(N)
      D3N=D3(N)
      GO TO 86
   85 DQTEN=DQTE(N)
      TE1=GCSTE*DQTEN
      TE2=GC2TE*DQTEN
      FN1=N1
      PNG=FN1*PGC
   86 DO 22 Q=1,NMAX
      P2=1
      Q1=Q-1
      IF(Q1.EQ.0) P2=2
      P3=MM(Q)
      W8=Q1*FN1B
      NQEO=NEO+Q1-2*(Q1/2)
      NEQQ=2
      IF(N1.EQ.Q1.AND.Q1.NE.0) NEQQ=1
      QPN=2
      IF(Q1.EQ.0.AND.N1.EQ.0) QPN=1
      GO TO (31,32), ITM
   31 IN=N1-Q1
      IP=N1+Q1
      TMMQ=TMM(Q)
      TMPQ=TMP(Q)
      FTM=FXY(IP,TMPN,TMMQ)-FXY(IN,TMMN,-TMMQ)
     1-FXY(IP,TMMN,TMPQ)+FXY(IN,TMPN,-TMPQ)
      Z1=W1*(FTM+GTM(Q)*DTMN)
      GO TO 47
   32 GO TO (46,107,117), NQEO
   46 GO TO (118,119), QPN
```

```
  118 Z1R=ZZTM
      GO TO 120
  119 Z1R=ZEETM
      GO TO 120
  107 Z1R=ZOETM
      GO TO 120
  117 Z1R=ZOOTM
  120 Z1R=Z1R*TM1*DQTM(Q)
      GO TO (121,122), NEQQ
  121 Z1R=Z1R-TM2
  122 Z1R=-W1*Z1R
   47 GO TO (36,37) ITE
   36 GTEQ=GTE(Q)
      G4Q=G4(Q)
      IN=N1-Q1
      IP=N1+Q1
      TENQ=TEN(Q)
      TEPQ=TEP(Q)
      F1=FXY(IN,TENN,-TENQ)
      F2=FXY(IP,TEPN,TENQ)
      F6=FXY(IP,TENN,TEPQ)
      F7=FXY(IN,TEPN,-TEPQ)
      F8=F2-F1
      F9=F2+F1
      F1=F6-F7
      F2=F6+F7
      FTE=F8-F1
      F3=F9+F2
      F4=F8+F1
      F5=F9-F2
      Z2=W2*(FTE*GTEQ*GTEN)
      Z3=W3*(F3*GTEQ*G3N)
      Z4=-W3*(F4*G4Q*GTEN)
      Z5=W5*(F5*G4Q*G3N)
      IF(N1.EQ.Q1.AND.N1.NE.0) Z5=W6*Z5
      GO TO 49
   37 GO TO (123,124,125), NOED
  123 GO TO (126,127), QPE
  126 ZR=ZZTE
      GO TO 128
  127 ZR=ZEETE
      GO TO 128
  124 ZR=ZOETE
      GO TO 128
```

```
125 ZR=ZOOTE
128 ZR=ZR*TE1*DQTE(Q)
    Z2R=ZR
    GO TO (129,130), NEQQ
129 Z2R=Z2R-TE2
130 Z3R=PNG*Z2R
    FQ1=Q1
    PQG=FQ1*PGC
    Z4R=PQG*Z2R
    Z5R=PNG*PQG*ZR
    GO TO (131,132), NEQQ
131 Z5R=Z5R+TE2
132 Z2R=W2*Z2R
    Z3R=W3*Z3R
    Z4R=W3*Z4R
    Z5R=-W5*Z5R
    GO TO (133,49) NEQQ
133 Z5R=W6+Z5R
 49 IF((ITM+ITE).NE.3) GO TO 87
    GO TO (53,54),ITM
 53 Z2=Z2R
    Z3=Z3R
    Z4=Z4R
    Z5=Z5R
    GO TO 87
 54 Z1=Z1R
 87 NTM=NTM
    NTE=NTE
    DO 23 N=N2,N3
    KMN=1
    N1=N-1
    IF(M1.EQ.0.OR.N1.EQ.0) GO TO 26
    KMN=2
    NTM=NTM+1
 26 NTE=NTE+1
    W9=N1*Q1
    FN1B=N1/BC
    TB=TA/BMN(NTE)
    PTM=QTM
    PTE=QTE
    DO 24 P=P2,P3
    KPQ=1
    P1=P-1
    IF(Q1.EQ.0.OR.P1.EQ.0) GO TO 27
```

```
      KPQ=2
      PTM=PTM+1
   27 PTE=PTE+1
      W10=N1*P1
      W11=P1*PM1B
      T1=TB/BMN(PTE)
      IF(M1.EQ.0) T1=T1*SQ2
      IF(N1.EQ.0) T1=T1*SQ2
      IF(P1.EQ.0) T1=T1*SQ2
      IF(Q1.EQ.0) T1=T1*SQ2
      PE1P=PE1(P)
      PE2P=PE2(P)
      PE3P=PE3(P)
      PE4P=PE4(P)
      KB=1+(KMN+KPQ)/4
      KM=(PTM-1)*K2
      KE1=KTM+NTE
      KMM=KM+NTM
      KEM=KM+KE1
      KE2=(KTM+PTE-1)*K2
      KME=KE2+NTM
      KEE=KE2+KE1
      K=0
      NJ=N
      DO 28 J=1,2
      PE1NJ=PE1(NJ)
      PE2NJ=PE2(NJ)
      PE3NJ=PE3(NJ)
      PE4NJ=PE4(NJ)
      PAG1=PE2P*PE3NJ-PE1P*PE4NJ
      PAG2=PE2P*PE2NJ+PE1P*PE1NJ
      PAG3=PE4P*PE1NJ-PE3P*PE2NJ
      PAG4=PE4P*PE4NJ+PE3P*PE3NJ
      IF(J.EQ.1) GO TO 30
      PAG1=-PAG1
      PAG4=-PAG4
   30 IF((ITE+ITM).EQ.4) GO TO 56
   50 S1=PAG2*(Z1-Z2)
      S3=PAG1*Z3
      S4=PAG3*Z4
      S5=PAG4*Z5
   64 GO TO (33,38), KB
   38 IMM=KMM+K
      YMM=T1*(W8*S1+W9*S3-W10*S4-W11*S5)
```

```
      Y(IMM)=Y(IMM)+YMM
   33 GO TO (34,39),KPQ
   39 IEM=KEM+K
      YEM=T1*(W9*S1-W8*S3-W11*S4+W10*S5)
      Y(IEM)=Y(IEM)+YEM
   34 GO TO (35,40),KMN
   40 IME=KME+K
      YME=T1*(W10*S1+W11*S3+W8*S4+W9*S5)
      Y(IME)=Y(IME)+YME
   35 IEE=KEE+K
      YEE=T1*(W11*S1-W10*S3+W9*S4-W8*S5)
      Y(IEE)=Y(IEE)+YEE
      GO TO 57
   56 S1R=PAG2*(Z1R-Z2R)
      S3R=PAG1*Z3R
      S4R=PAG3*Z4R
      S5R=PAG4*Z5R
      GO TO (58,59), KB
   59 IMM=KMM+K
      YMM=T1*(W8*S1R+W9*S3R-W10*S4R-W11*S5R)
      Y(IMM)=Y(IMM)+YMM
   58 GO TO (60,61), KPQ
   61 IEM=KEM+K
      YEM=T1*(W9*S1R-W8*S3R-W11*S4R+W10*S5R)
      Y(IEM)=Y(IEM)+YEM
   60 GO TO (62,63), KMN
   63 IME=KME+K
      YME=T1*(W10*S1R+W11*S3R+W8*S4R+W9*S5R)
      Y(IME)=Y(IME)+YME
   62 IEE=KEE+K
      YEE=T1*(W11*S1R-W10*S3R+W9*S4R-W8*S5R)
      Y(IEE)=Y(IEE)+YEE
   57 K=K1
      MJ=M+PMAX
   28 CONTINUE
   24 CONTINUE
   23 CONTINUE
      QTM=PTM
      QTE=PTE
   22 CONTINUE
      NTM=MTM
      NTE=MTE
   21 CONTINUE
   20 CONTINUE
```

```
   19 CONTINUE
      WRITE(21,44)
   44 FORMAT('R IS TOO LARGE')
      STOP
   25 IF(R.NE.1) GO TO 41
      WRITE(21,42)
   42 FORMAT ('XN IS TOO SMALL')
      STOP
   41 K4=K1*K2
      J1=0
      DO 89 J=1,K1
      DO 88 I=1,K1
      J1=J1+1
      J2=J1+K1
      J3=J1+K4
      J4=J3+K1
      Y(J3)=Y(J2)
      Y(J4)=Y(J1)
   88 CONTINUE
      J1=J1+K1
   89 CONTINUE
      K5=K2+1
      IY=1
      DO 43 I=1,K2
      Y(IY)=Y(IY)+YREC(I)
      IY=IY+K5
   43 CONTINUE
      DO 191 R=1,RTM
      SMAX=STM(R)
      IF(SMAX.EQ.0) GO TO 191
      DO 192 S=1,SMAX
      BTME(S,R)=0.
      BTMO(S,R)=0.
  192 CONTINUE
  191 CONTINUE
      DO 193 R=1,RTE
      SMAX=STE(R)
      IF(SMAX.EQ.0) GO TO 193
      DO 194 S=1,SMAX
      BTEE(S,R)=0.
      BTEO(S,R)=0.
  194 CONTINUE
  193 CONTINUE
      READ(20,109) RTMR,RTER
```

```
109 FORMAT(10I4)
    WRITE(21,66) RTMR,RTER
 66 FORMAT('RTMR=',I4,', RTER=',I4)
    READ(20,109)(STMR(R),R=1,RTMR)
    WRITE(21,79)(STMR(R),R=1,RTMR)
 79 FORMAT('STMR'/(10I4))
    READ(20,109)(STER(R),R=1,RTER)
    WRITE(21,55)(STER(R),R=1,RTER)
 55 FORMAT('STER'/(10I4))
    DO 156 R=1,RTMR
    SMAX=STMR(R)
    IF(SMAX.EQ.0) GO TO 156
    R1=R-1
    READ(20,164)(BTME(S,R),S=1,SMAX)
164 FORMAT(4D14.7)
    WRITE(21,80) R,(BTME(S,R), S=1,SMAX)
 80 FORMAT('BTME(',I4,',S)'/(4D14.7))
    IF(R1.EQ.0) GO TO 156
    READ(20,164)(BTMO(S,R),S=1,SMAX)
    WRITE(21,163) R,(BTMO(S,R), S=1,SMAX)
163 FORMAT('BTMO(',I4,',S)'/(4D14.7))
156 CONTINUE
    DO 167 R=1,RTER
    SMAX=STER(R)
    IF(SMAX.EQ.0) GO TO 167
    R1=R-1
    READ(20,164)(BTEE(S,R),S=1,SMAX)
    WRITE(21,184) R,(BTEE(S,R), S=1,SMAX)
184 FORMAT('BTEE(',I4,',S)'/(4D14.7))
    IF(R1.EQ.0) GO TO 167
    READ(20,164)(BTEO(S,R),S=1,SMAX)
    WRITE(21,81) R,(BTEO(S,R), S=1,SMAX)
 81 FORMAT('BTEO(',I4,',S)'/(4D14.7))
167 CONTINUE
    WRITE(21,185) RTM,RTE
185 FORMAT('RTM=',I4,', RTE='I4)
    WRITE(21,186)(STM(R),R=1,RTM)
186 FORMAT('STM'/(10I4))
    WRITE(21,187)(STE(R),R=1,RTE)
187 FORMAT('STE'/(10I4))
    NTM=0
    NTE=0
    DO 16 N=1,NMAX
    M2=1
```

```
      N1=N-1
      FN1=N1
      IF(N1.EQ.0) M2=2
      M3=MM(N)
      DO 74 M=M2,M3
      M1=M-1
      FM1=M1
      FM1=FM1/BC
      X1TM=0.
      X1TE=0.
      X2TM=0.
      X2TE=0.
      SGR=1.
      NTE=NTE+1
      JTME=NTE
      JTMO=NTE
      DO 72 R=1,RTM
      SGR=-SGR
      SMAX=STM(R)
      IF(SMAX.EQ.0) GO TO 72
      R1=R-1
      DO 73 S=1,SMAX
      Z1=ITME(JTME)*BTME(S,R)
      JTME=JTME+KTE
      W1=SGR*FN1
      W2=SGR*FM1
      X1TM=-W1*Z1+X1TM
      X1TE=-W2*Z1+X1TE
      X2TM=FN1*Z1+X2TM
      X2TE=FM1*Z1+X2TE
      IF(R1.EQ.0) GO TO 73
      Z1=ITMO(JTMO)*BTMO(S,R)
      JTMO=JTMO+KTE
      X1TM=W1*Z1+X1TM
      X1TE=W2*Z1+X1TE
      X2TM=FN1*Z1+X2TM
      X2TE=FM1*Z1+X2TE
   73 CONTINUE
   72 CONTINUE
      SGR=1.
      JTEE=NTE
      JTEO=NTE
      DO 168 R=1,RTE
      SGR=-SGR
```

```
      SMAX=STE(R)
      IF(SMAX.EQ.0) GO TO 168
      R1=R-1
      DO 169 S=1,SMAX
      W5=ITEEP(JTEE)
      W6=ITEEZ(JTEE)
      JTEE=JTEE+KTE
      W3=FN1*W5+FN1*W6
      W4=FN1*W5-FN1*W6
      W1=SGR*W3
      W2=SGR*W4
      Z1=BTEE(S,R)
      X1TM=W1*Z1+X1TM
      X1TE=W2*Z1+X1TE
      X2TM=W3*Z1+X2TM
      X2TE=W4*Z1+X2TE
      IF(R1.EQ.0) GO TO 169
      W5=ITEOP(JTEO)
      W6=ITEOZ(JTEO)
      JTEO=JTEO+KTE
      W3=FN1*W5-FN1*W6
      W4=FN1*W5+FN1*W6
      W1=SGR*W3
      W2=SGR*W4
      Z1=BTEO(S,R)
      X1TM=W1*Z1+X1TM
      X1TE=W2*Z1+X1TE
      X2TM=-W3*Z1+X2TM
      X2TE=-W4*Z1+X2TE
  169 CONTINUE
  168 CONTINUE
      IF(N1.EQ.0.OR.M1.EQ.0) GO TO 171
      NTM=NTM+1
      TI(NTM)=-U*X1TM
      J1=NTM+K1
      TI(J1)=-U*X2TM
  171 J1=NTE+KTM
      TI(J1)=-U*X1TE
      J2=J1+K1
      TI(J2)=-U*X2TE
   74 CONTINUE
   16 CONTINUE
      WRITE(21,204)(TI(I),I=1,K2)
  204 FORMAT('TI'/(4E14.7))
```

```
      CALL DECOMP(K2,IPS,Y)
      CALL SOLVE(K2,IPS,Y,TI,V)
      WRITE(21,206)(V(I),I=1,K2)
  206 FORMAT('V'/(4E14.7))
      BET=DSQRT(BKB2-BMN2(1))/B
      ARG=BET*(L1-XZ)
      CS=DCOS(ARG)
      SN=DSIN(ARG)
      KV=KTM+1
      S1=.5D+0*DSQRT(BET/BKA)
      SA=S1*V(KV)/(CS*ZL1+U*SN)
      C1OUT=(ZL1+1.D+0)*DCMPLX(CS,SN)*SA
      C1IN=(ZL1-1.D+0)*DCMPLX(CS,-SN)*SA
      WRITE(21,69) C1OUT,C1IN
   69 FORMAT('C1OUT=',2E14.7,', C1IN=',2E14.7)
      ARG=BET*(L2-XZ)
      CS=DCOS(ARG)
      SN=DSIN(ARG)
      KV=KV+K1
      SA=S1*V(KV)/(CS*ZL2+U*SN)
      C2OUT=(ZL2+1.D+0)*DCMPLX(CS,SN)*SA
      C2IN=(ZL2-1.D+0)*DCMPLX(CS,-SN)*SA
      WRITE(21,70) C2OUT,C2IN
   70 FORMAT('C2OUT=',2E14.7,', C2IN=',2E14.7)
      C1OUTS=C1OUT*CONJG(C1OUT)
      C1INS=C1IN*CONJG(C1IN)
      C2OUTS=C2OUT*CONJG(C2OUT)
      C2INS=C2IN*CONJG(C2IN)
      PT=C1OUTS-C1INS+C2OUTS-C2INS
      WRITE(21,82) C1OUTS,C1INS,C2OUTS,C2INS,PT
   82 FORMAT('C1OUTS=',E14.7,', C1INS=',E14.7,', C2OUTS=',
     1E14.7/'C2INS=',E14.7,', PT=',E14.7)
      W7=0.
      SGR=1.
      JTME=0
      JTMO=0
      PRM=0.
      PR=0.
      PIN=0.
      DO 172 K=1,KTM
      SGR=-SGR
      SMAX=STM(K)
      IF(SMAX.EQ.0) GO TO 172
      K1=K-1
```

```
      DO 173 S=1,SMAX
      Z1=BTME(S,R)
      W1=Z1*CONJG(Z1)
      Z2=BTMO(S,R)
      W2=Z2*CONJG(Z2)
      W7=W7+(W1+W2)*BTM(S,R)
      XTME=0.
      XTMO=0.
      NTM=0
      NTE=0
      DO 174 N=1,NMAX
      M2=1
      N1=N-1
      FN1=N1
      IF(N1.EQ.0) M2=2
      M3=MM(N)
      DO 176 M=M2,M3
      M1=M-1
      FM1=M1
      FM1=FM1/BC
      JTME=JTME+1
      W5=ITME(JTME)
      IF(M1.EQ.0.OR.N1.EQ.0) GO TO 178
      NTM=NTM+1
      J1=NTM+K1
      Z1=V(NTM)
      Z3=V(J1)
      W3=FN1*W5
      W1=SGR*W3
      XTME=-W1*Z1+W3*Z3+XTME
  178 NTE=NTE+1
      J2=NTE+KTM
      J3=J2+K1
      Z2=V(J2)
      Z4=V(J3)
      W4=FM1*W5
      W2=SGR*W4
      XTME=-W2*Z2+W4*Z4+XTME
      IF(R1.EQ.0) GO TO 176
      JTMO=JTMO+1
      W5=ITMO(JTMO)
      IF(M1.EQ.0.OR.N1.EQ.0) GO TO 179
      W3=FN1*W5
      W1=SGR*W3
```

```
      XTMO=W1*Z1+W3*Z3+XTMO
  179 W4=FN1*W5
      W2=SGR*W4
      XTMO=W2*Z2+W4*Z4+XTMO
  176 CONTINUE
  174 CONTINUE
      XTME=.5*XTME
      BT=BTME(S,R)
      PINS=BT*CONJG(BT)
      PIN=PIN+PINS
      CTME=-BT+XTME
      XTMES=XTME*CONJG(XTME)
      CTMES=CTME*CONJG(CTME)
      PRM=PRM+XTMES
      PR=PR+CTMES
      WRITE(21,160) R,S
  160 FORMAT('R=',I4,', S=',I4)
      WRITE(21,161) XTME,XTMES
  161 FORMAT('XTME=',2E14.7,', XTMES=',E14.7)
      WRITE(21,162) CTME,CTMES
  162 FORMAT('CTME=',2E14.7,', CTMES=',E14.7)
      IF(R1.EQ.0) GO TO 173
      XTMO=.5*XTMO
      BT=BTMO(S,R)
      PINS=BT*CONJG(BT)
      PIN=PIN+PINS
      CTMO=-BT+XTMO
      XTMOS=XTMO*CONJG(XTMO)
      CTMOS=CTMO*CONJG(CTMO)
      PRM=PRM+XTMOS
      PR=PR+CTMOS
      WRITE(21,157) XTMO,XTMOS
  157 FORMAT('XTMO=',2E14.7,', XTMOS=',E14.7)
      WRITE(21,158) CTMO,CTMOS
  158 FORMAT('CTMO=',2E14.7,', CTMOS=',E14.7)
  173 CONTINUE
  172 CONTINUE
      SGR=1.
      JTME=0
      JTMO=0
      DO 180 R=1,RTE
      SGR=-SGR
      SMAX=STE(R)
      IF(SMAX.EQ.0) GO TO 180
```

```
      R1=R-1
      DO 181 S=1,SMAX
      Z1=BTEE(S,R)
      W1=Z1*CONJG(Z1)
      Z2=BTEO(S,R)
      W2=Z2*CONJG(Z2)
      W7=W7+(W1+W2)*BTE(S,R)
      XTEE=0.
      XTEO=0.
      NTM=0
      NTE=0
      DO 182 N=1,NMAX
      M2=1
      N1=N-1
      FN1=N1
      IF(N1.EQ.0) M2=2
      M3=MN(N)
      DO 75 M=M2,M3
      M1=M-1
      FM1=M1
      FM1=FM1/BC
      JTEE=JTEE+1
      W5=ITEEP(JTEE)
      W6=ITEEZ(JTEE)
      IF(M1.EQ.0.OR.N1.EQ.0) GO TO 175
      NTM=NTM+1
      J1=NTM+K1
      Z1=V(NTM)
      Z3=V(J1)
      W3=FN1*W5+FM1*W6
      W1=SGR*W3
      XTEE=W1*Z1+W3*Z3+XTEE
  175 NTE=NTE+1
      J2=NTE+KTM
      J3=J2+K1
      Z2=V(J2)
      Z4=V(J3)
      W4=FM1*W5-FN1*W6
      W2=SGR*W4
      XTEE=W2*Z2+W4*Z4+XTEE
      IF(R1.EQ.0) GO TO 75
      JTEO=JTEO+1
      W5=ITEOP(JTEO)
      W6=ITEOZ(JTEO)
```

```
      IF(M1.EQ.0.OR.N1.EQ.0) GO TO 183
      W3=FN1*W5-FM1*W6
      W1=SGR*W3
      XTEO=W1*Z1-W3*Z3+XTEO
  183 W4=FM1*W5+FN1*W6
      W2=SGR*W4
      XTEO=W2*Z2-W4*Z4+XTEO
   75 CONTINUE
  182 CONTINUE
      XTEE=.5*XTEE
      BT=BTEE(S,R)
      PINS=BT*CONJG(BT)
      PIN=PIN+PINS
      CTEE=-BT+XTEE
      XTEES=XTEE*CONJG(XTEE)
      CTEES=CTEE*CONJG(CTEE)
      PRM=PRM+XTEES
      PR=PR+CTEES
      WRITE(21,160) R,S
      WRITE(21,159) XTEE,XTEES
  159 FORMAT('XTEE=',2E14.7,', XTEES=',E14.7)
      WRITE(21,165) CTEE, CTEES
  165 FORMAT('CTEE=',2E14.7,', CTEES=',E14.7)
      IF(R1.EQ.0) GO TO 181
      XTEO=.5*XTEO
      BT=BTEO(S,R)
      PINS=BT*CONJG(BT)
      PIN=PIN+PINS
      CTEO=-BT+XTEO
      XTEOS=XTEO*CONJG(XTEO)
      CTEOS=CTEO*CONJG(CTEO)
      PRM=PRM+XTEOS
      PR=PR+CTEOS
      WRITE(21,166) XTEO,XTEOS
  166 FORMAT('XTEO=',2E14.7,', XTEOS=',E14.7)
      WRITE(21,90) CTEO,CTEOS
   90 FORMAT('CTEO=',2E14.7,', CTEOS=',E14.7)
  181 CONTINUE
  160 CONTINUE
      WRITE(21,190) PIN,PRM,PR
  190 FORMAT('PIN=',E14.7,' PRM=',E14.7,' PR=',E14.7)
      BKAPLT(KA)=BKA
      PINC(KA)=PIN
      PTRAN(KA)=PT
```

```
      PREFL(KA)=PR
      PTOTAL=PT+PR
      WRITE(21,154) PTOTAL
  154 FORMAT('PTOTAL=',E14.7)
      YHM=0
      YHE=0
      DO 91 I=1,K2
      YEE=CONJG(V(I))
      YHM=YHM+YREC(I)*V(I)*YEE
      YHE=YHE+TI(I)*YEE
   91 CONTINUE
      PTA=-AIMAG(YHM)
      PRMA=-PTA-AIMAG(YHE)
      WRITE(21,93) PTA,PRMA
   93 FORMAT('PTA=',E14.7,', PRMA=',E14.7)
      IF(KA.NE.KE3(KAE)) GO TO 48
      KAE=KAE+1
      DEL=PI/(NPHI-1)
      DO 105 J=1,NPHI
      E3A1P(J)=0.
      E3A2P(J)=0.
      FJ1=J-1
      FJ1D=FJ1*DEL
      PHI1(J)=PI-FJ1D
      PHI2(J)=FJ1D
  105 CONTINUE
      DEL=PI/(NZ-1)
      DO 106 J=1,NZ
      E3A1Z(J)=0.
      E3A2Z(J)=0.
      FJ1=J-1
      Z(J)=FJ1*DEL
  106 CONTINUE
      IF(KAE.NE.2) GO TO 155
      WRITE(21,152)(PHI2(I),I=1,NPHI)
  152 FORMAT('PHI2'/(5E14.7))
      WRITE(21,151)(Z(I),I=1,NZ)
  151 FORMAT('Z'/(5E14.7))
  155 SA=-PI2*DSQRT(PI*BC/W7)
      DO 136 J=1,PMAX
      FJ1=J-1
      SINP(J)=DSIN(FJ1*PI5)*SA
      SINQ(J)=SINP(J)*Z2
  136 CONTINUE
```

```
      JTM=0
      JTE=0
      DO 101 Q=1,NMAX
      Q1=Q-1
      FQ1=Q1
      FQ1C=FQ1/C
      P2=1
      IF(Q1.EQ.0) P2=2
      P3=MM(Q)
      SINQQ=SINQ(Q)
      DO 104 P=P2,P3
      JTE=JTE+1
      JTE1=JTE+KTM
      JTE2=JTE1+K1
      BMNJ=1./BMN(JTE)
      P1=P-1
      FP1=P1
      IF(Q1.EQ.0) BMNJ=BMNJ*SQ2
      IF(P1.EQ.0) BMNJ=BMNJ*SQ2
      SINPP=SINP(P)
      BMNJP=BMNJ*FP1/B
      BMNJQ=BMNJ*FQ1C
      BMNJPP=BMNJP*SINPP
      BMNJQQ=BMNJQ*SINQQ
      YMM=BMNJQQ*V(JTE1)
      YEM=BMNJPP*V(JTE1)
      YME=-BMNJQQ*V(JTE2)
      YEE=BMNJPP*V(JTE2)
      IF(P1.EQ.0.OR.Q1.EQ.0) GO TO 134
      JTM=JTM+1
      JTM2=JTM+K1
      BMNJPQ=BMNJP*SINQQ
      BMNJQP=BMNJQ*SINPP
      YMM=-BMNJPQ*V(JTM)+YMM
      YEM=BMNJQP*V(JTM)+YEM
      YME=BMNJPQ*V(JTM2)+YME
      YEE=BMNJQP*V(JTM2)+YEE
  134 PT=FP1
      DO 135 J=1,NPHI
      E3A1P(J)=E3A1P(J)+YMM*COS(PT*PHI1(J))
      E3A2P(J)=E3A2P(J)+YME*COS(PT*PHI2(J))
  135 CONTINUE
      PT=FQ1
      DO 148 J=1,NZ
```

```
      PR=COS(PT*Z(J))
      E3A1Z(J)=E3A1Z(J)+YEM*PR
      E3A2Z(J)=E3A2Z(J)+YEE*PR
  148 CONTINUE
  104 CONTINUE
  101 CONTINUE
      DO 137 J=1,NPHI
      E3A1PS(J)=CABS(E3A1P(J))
      E3A2PS(J)=CABS(E3A2P(J))
  137 CONTINUE
      DO 138 J=1,NZ
      E3A1ZS(J)=CABS(E3A1Z(J))
      E3A2ZS(J)=CABS(E3A2Z(J))
  138 CONTINUE
      WRITE(21,140)(E3A1PS(J),J=1,NPHI)
  140 FORMAT('E3A1PS'/(5E14.7))
      WRITE(21,141)(E3A1ZS(J),J=1,NZ)
  141 FORMAT('E3A1ZS'/(5E14.7))
      WRITE(21,142)(E3A2PS(J),J=1,NPHI)
  142 FORMAT('E3A2PS'/(5E14.7))
      WRITE(21,143)(E3A2ZS(J),J=1,NZ)
  143 FORMAT('E3A2ZS'/(5E14.7))
   48 CONTINUE
      WRITE(21,149)(BKAPLT(I),I=1,KAN)
  149 FORMAT('BKAPLT'/(5E14.7))
      WRITE(21,189)(PINC(I),I=1,KAN)
  189 FORMAT('PINC'/(5E14.7))
      WRITE(21,92)(PTRAN(I),I=1,KAN)
   92 FORMAT('PTRAN'/(5E14.7))
      WRITE(21,139)(PREFL(I),I=1,KAN)
  139 FORMAT('PREFL'/(5E14.7))
      STOP
      END
```

Appendix A

Verification of (3.119)

The term in parentheses on the right-hand side of (3.119) is given by eqs. (4.8) and (4.9) of [2]:[†]

$$-j\hat{I}_i^{\alpha\mathrm{TM}} e^{j\beta_{01}^{\mathrm{TM}} L_3} = -j\left(\frac{8n\phi_o}{k_{mn}b}\right)\sqrt{\frac{b}{\pi c}}\left[\hat{G}_n^{\mathrm{TM}}\right]_{01},\left\{\begin{array}{ll} 0, & m \text{ even} \\ \frac{1}{m}, & m \text{ odd}\end{array}\right\} \quad (A.1)$$

$$-j\hat{I}_i^{\alpha\mathrm{TE}} e^{j\beta_{01}^{\mathrm{TM}} L_3} = -j\left(\frac{8\phi_o}{k_{mn}b}\right)\sqrt{\frac{\epsilon_n c}{2\pi b}}\left[\hat{G}_n^{\mathrm{TM}}\right]_{01},\left\{\begin{array}{ll} 0, & m \text{ even} \\ 1, & m \text{ odd}\end{array}\right\} \quad (A.2)$$

where $\left[\hat{G}_n^{\mathrm{TM}}\right]_{01}$ is $\hat{G}_n^{\mathrm{TM}}$ when $r = 0$ and $s = 1$. The left-hand side of (3.119) is given by (3.116):

$$-j\sqrt{\eta}I_i^{\alpha\beta} = -jI_i^{\alpha\beta,\mathrm{TMe}},\left\{\begin{array}{l} \alpha = 1,2 \\ \beta = \mathrm{TM, TE}\end{array}\right. \quad (A.3)$$

where it is understood that $I_i^{\alpha\beta,\mathrm{TMe}}$ is evaluated at $(r,s) = (0,1)$. In Appendix A, we verify (3.119) by showing that the right-hand side of (A.3) is equal to the product of $\sqrt{k/\beta_{01}^{\mathrm{TM}}}$ with the right-hand sides of (A.1) and (A.2).

Substituting (3.110) into (3.57), (3.58), (3.63), and (3.64) and multiplying

[†]The right-hand side of eq. (4.8) of [2] is a poor expression because it is indeterminate when $m = 0$. However, the right-hand side of (4.8) of [2] should, as is evident from its derivation in [2], vanish when $m = 0$.

the resulting equations by $-j$, we express the right-hand side of (A.3) as

$$-jI_i^{\alpha\mathrm{TM,TM}e} = -jnT^{\mathrm{TM}}\phi_m^{b2}\left[\hat{G}_n^{\mathrm{TM}}\right]_{01}, \quad \alpha = 1,2 \tag{A.4}$$

$$-jI_i^{\alpha\mathrm{TE,TM}e} = -j\left(\frac{mc}{b}\right)T^{\mathrm{TM}}\phi_m^{b2}\left[\hat{G}_n^{\mathrm{TM}}\right]_{01}, \quad \alpha = 1,2 \tag{A.5}$$

where T^{TM} is given by (3.60) with $r = 0$ and $s = 1$. In (A.4) and (A.5), ϕ_m^{b2} is given by eq. (6.29) of [2] with p replaced by m and with $r = 0$:

$$\phi_m^{b2} = \phi_m^{(2)}. \tag{A.6}$$

Setting $r = 0$ and $p = m$ in eq. (3.41) of [2] and using eqs. (3.44) and (3.45) of [2], we have†

$$\phi_m^{(2)} = \frac{2b}{\pi x_o}\left\{\begin{array}{ll} 0, & m \text{ even} \\ \frac{1}{m}, & m \text{ odd} \end{array}\right\}. \tag{A.7}$$

In view of (3.89), substitution of (A.7) into (A.6) gives

$$\phi_m^{b2} = \frac{4\phi_o}{\pi}\left\{\begin{array}{ll} 0, & m \text{ even} \\ \frac{1}{m}, & m \text{ odd} \end{array}\right\}. \tag{A.8}$$

Substitution of (3.60) and (A.8) into (A.4) and (A.5) gives

$$-jI_i^{\alpha\mathrm{TM,TM}e} = -j\left(\frac{8n\phi_o}{k_{mn}b}\right)\sqrt{\left(\frac{b}{\pi c}\right)\left(\frac{k}{\beta_{01}^{\mathrm{TM}}}\right)}\left[\hat{G}_n^{\mathrm{TM}}\right]_{01}\left\{\begin{array}{ll} 0, & m \text{ even} \\ \frac{1}{m}; & m \text{ odd} \end{array}\right\} \tag{A.9}$$

$$-jI_i^{\alpha\mathrm{TE,TM}e} = -j\left(\frac{8\phi_o}{k_{mn}b}\right)\sqrt{\left(\frac{\epsilon_n c}{2\pi b}\right)\left(\frac{k}{\beta_{01}^{\mathrm{TM}}}\right)}\left[\hat{G}_n^{\mathrm{TM}}\right]_{01}\left\{\begin{array}{ll} 0, & m \text{ even} \\ 1, & m \text{ odd} \end{array}\right\}. \tag{A.10}$$

Comparing (A.9) and (A.10) with (A.1) and (A.2), we see that expressions (A.9) and (A.10) for the right-hand side of (A.3) are indeed equal to the product of $\sqrt{k/\beta_{01}^{\mathrm{TM}}}$ with the right-hand sides of (A.1) and (A.2).

†It is evident from eq. (E.15) of [1] that $\phi_m^{(2)}$ should vanish when $m = 0$.

Appendix B

Verification of the Equality of Modal Coefficients

In Appendix B, we show that expressions (5.21), (5.31), and (5.32) for the modal coefficients $C_{01}^{\mathrm{TM}e}$, $C_{11}^{\mathrm{TE}e}$, and $C_{11}^{\mathrm{TE}o}$ reduce to the right-hand sides of eqs. (6.88), (6.96), and (6.99) of [2] when $B_{01}^{\mathrm{TM}e} = 1$ is the only nonzero B in (3.1) and when only the TM_{01} and TE_{11} modes can propagate in the circular waveguide. The V's in eqs. (6.88), (6.96), and (6.99) of [2] are the products of $\sqrt{Z_{01}^{\mathrm{TM}eo}}e^{-j\beta_{01}^{\mathrm{TM}}L_3}$ with the V's in the present report because, as stated in Section 3.5, the excitation in the present report is that of [2] multiplied by $e^{j\beta_{01}^{\mathrm{TM}}L_3}/\sqrt{Z_{01}^{\mathrm{TM}eo}}$. Multiplying each V in eqs. (6.88), (6.96), and (6.99) of [2] by $\sqrt{Z_{01}^{\mathrm{TM}eo}}\,e^{-j\beta_{01}^{\mathrm{TM}}L_3}$, using (3.27) to replace this multiplicative factor by $\sqrt{\eta\beta_{01}^{\mathrm{TM}}/k}\,e^{-j\beta_{01}^{\mathrm{TM}}L_3}$, and using eqs. (6.89), (6.90), (6.97), (6.98), (6.100), and (6.101) of [2],† we convert expressions (6.88), (6.96), and (6.99) of [2] to expressions in terms of the V's in the present report:

$$C_{01}^{\mathrm{TM}e} = -1 + \sqrt{\left(\frac{\pi b}{c}\right)\left(\frac{k}{\beta_{01}^{\mathrm{TM}}}\right)}\sum_{\gamma=1}^{2}\sum_{q=0}\sum_{\substack{p=0\\p+q\neq 0}}\epsilon_{pq}\left(q\left(\frac{V_{pq}^{\gamma\mathrm{TM}}}{\sqrt{\eta}}\right)\right.$$

†There are minor errors in eqs. (6.97), (6.98), (6.100), and (6.101) of [2]. The right-hand sides of these equations should be $C_{11,pq}^{\mathrm{TE}e,\gamma\mathrm{TM}}$, $C_{11,pq}^{\mathrm{TE}e,\gamma\mathrm{TE}}$, $C_{11,pq}^{\mathrm{TE}o,\gamma\mathrm{TM}}$, and $C_{11,pq}^{\mathrm{TE}o,\gamma\mathrm{TE}}$, respectively.

$$+\left(\frac{pc}{b}\right)\left(\frac{V_{pq}^{\gamma\mathrm{TE}}}{\sqrt{\eta}}\right)\Bigg)\left[\hat{G}_q^{\mathrm{TM}}\right]_{01}\phi_p^{(2)} \tag{B.1}$$

$$C_{11}^{\mathrm{TE}e} = \sqrt{\left(\frac{2\pi b}{c}\right)\left(\frac{\beta_{11}^{\mathrm{TE}}}{k(x_{11}'^2-1)}\right)}\sum_{\gamma=1}^{2}\sum_{q=0}\sum_{\substack{p=0\\p+q\neq 0}}\epsilon_{pq}$$

$$\cdot\Bigg\{\left(q\left[\hat{G}_q^{\mathrm{TE}}\right]_{11}\phi_p^{b1}+\left(\frac{pc}{b}\right)\left(\frac{\sin\phi_o}{\phi_o}\right)\frac{x_{11}'^2\left[\hat{G}_q^{(4)}\right]_{11}\phi_p^{b4}}{\beta_{11}^{\mathrm{TE}}a}\right)\left(\frac{V_{pq}^{\gamma\mathrm{TM}}}{\sqrt{\eta}}\right)$$

$$+\left(\left(\frac{pc}{b}\right)\left[\hat{G}_q^{\mathrm{TE}}\right]_{11}\phi_p^{b1}-q\left(\frac{\sin\phi_o}{\phi_o}\right)\frac{x_{11}'^2\left[\hat{G}_q^{(4)}\right]_{11}\phi_p^{b4}}{\beta_{11}^{\mathrm{TE}}a}\right)\left(\frac{V_{pq}^{\gamma\mathrm{TE}}}{\sqrt{\eta}}\right)\Bigg\} \tag{B.2}$$

$$C_{11}^{\mathrm{TE}o} = \sqrt{\left(\frac{2\pi b}{c}\right)\left(\frac{\beta_{11}^{\mathrm{TE}}}{k(x_{11}'^2-1)}\right)}\sum_{\gamma=1}^{2}\sum_{q=0}\sum_{\substack{p=0\\p+q\neq 0}}(-1)^{\gamma+1}\epsilon_{pq}$$

$$\cdot\Bigg\{\left(q\left[\hat{G}_q^{\mathrm{TE}}\right]_{11}\phi_p^{b2}-\left(\frac{pc}{b}\right)\left(\frac{\sin\phi_o}{\phi_o}\right)\frac{x_{11}'^2\left[\hat{G}_q^{(4)}\right]_{11}\phi_p^{b3}}{\beta_{11}^{\mathrm{TE}}a}\right)\left(\frac{V_{pq}^{\gamma\mathrm{TM}}}{\sqrt{\eta}}\right)$$

$$+\left(\left(\frac{pc}{b}\right)\left[\hat{G}_q^{\mathrm{TE}}\right]_{11}\phi_p^{b2}+q\left(\frac{\sin\phi_o}{\phi_o}\right)\frac{x_{11}'^2\left[\hat{G}_q^{(4)}\right]_{11}\phi_p^{b3}}{\beta_{11}^{\mathrm{TE}}a}\right)\left(\frac{V_{pq}^{\gamma\mathrm{TE}}}{\sqrt{\eta}}\right)\Bigg\}. \tag{B.3}$$

Now, the objective is to show that the right-hand sides of (5.21), (5.31), and (5.32) reduce to those of (B.1), (B.2), and (B.3) when $B_{01}^{\mathrm{TM}e} = 1$ is the only nonzero B in (3.1) and when only the TM_{01} and TE_{11} modes can propagate in the circular waveguide. Setting $(r,s) = (0,1)$ and $B_{rs}^{\mathrm{TM}e} = 1$ in (5.21), substituting the modified† right-hand sides of (3.57) and (3.63) for $\Gamma_j^{\gamma\mathrm{TM},\mathrm{TM}e}$, substituting the modified right-hand sides of (3.58) and (3.64) for $\Gamma_j^{\gamma\mathrm{TE},\mathrm{TM}e}$, and using

$$\phi_p^{b2} = \phi_p^{(2)}, \tag{B.4}$$

†Here, "modified" means with (m,n,i) replaced by (p,q,j). In (3.57), (3.58), (3.63), and (3.64), $\Gamma_{i,rs}^{\mathrm{TM}e}$ is given by (3.110) in which $\phi_m^{b2} = \phi_m^{(2)}$ and τ^{TM} is given by (3.60). Equation (3.60) contains ϵ_{pq} of eq. (6.60) of [2].

we obtain

$$C_{01}^{\mathrm{TMe}} = -1 + \sqrt{\left(\frac{\pi b}{c}\right)\left(\frac{k}{\beta_{01}^{\mathrm{TM}}}\right)} \sum_{\gamma=1}^{2} \sum_{q=0} \sum_{\substack{p=0 \\ p+q\neq 0}} \epsilon_{pq} \left(q \left(\frac{V_{pq}^{\gamma\mathrm{TM}}}{\sqrt{\eta}} \right) \right.$$

$$\left. + \left(\frac{pc}{b}\right)\left(\frac{V_{pq}^{\gamma\mathrm{TE}}}{\sqrt{\eta}}\right)\right) \left[\dot{G}_q^{\gamma\mathrm{TM}}\right]_{01} \phi_p^{(2)}. \tag{B.5}$$

Equation (B.4) was obtained by setting $r = 0$ in (5.17). Setting (r,s)=(1,1) and $B_{rs}^{\mathrm{TEe}} = 0$ in (5.31), substituting the modified right-hand sides of (3.71) and (3.79) for $\Gamma_j^{\mathrm{TM,TEe}}$, substituting the modified right-hand sides of (3.72) and (3.80) for $\Gamma_j^{\mathrm{TE,TEe}}$, and using (3.112), (3.113), (3.75), and eq. (6.60) of [2], we obtain

$$C_{11}^{\mathrm{TEe}} = \sqrt{\left(\frac{2\pi b}{c}\right)\left(\frac{\beta_{11}^{\mathrm{TE}}}{k(x_{11}'^2 - 1)}\right)} \sum_{\gamma=1}^{2} \sum_{q=0} \sum_{\substack{p=0 \\ p+q\neq 0}} \epsilon_{pq}$$

$$\cdot \left\{ \left(q \left[\dot{G}_q^{\mathrm{TE}}\right]_{11} \phi_p^{b1} + \left(\frac{pc}{b}\right)\left(\frac{\sin\phi_e}{\phi_e}\right) \frac{x_{11}'^2 \left[\dot{G}_q^{(4)}\right]_{11} \phi_p^{b4}}{\beta_{11}^{\mathrm{TE}} a} \right) \left(\frac{V_{pq}^{\gamma\mathrm{TM}}}{\sqrt{\eta}}\right) \right.$$

$$\left. + \left(\left(\frac{pc}{b}\right) \left[\dot{G}_q^{\mathrm{TE}}\right]_{11} \phi_p^{b1} - q \left(\frac{\sin\phi_e}{\phi_e}\right) \frac{x_{11}'^2 \left[\dot{G}_q^{(4)}\right]_{11} \phi_p^{b4}}{\beta_{11}^{\mathrm{TE}} a} \right) \left(\frac{V_{pq}^{\gamma\mathrm{TE}}}{\sqrt{\eta}}\right) \right\}. \tag{B.6}$$

Setting $(r,s) = (1,1)$ and $B_{rs}^{\mathrm{TEo}} = 0$ in (5.32), substituting the modified right-hand sides of (3.81) and (3.86) for $\Gamma_j^{\mathrm{TM,TEo}}$, substituting the modified right-hand sides of (3.82) and (3.87) for $\Gamma_j^{\mathrm{TE,TEo}}$, and using (3.114), (3.115), (3.75), and eq. (6.60) of [2], we obtain

$$C_{11}^{\mathrm{TEo}} = \sqrt{\left(\frac{2\pi b}{c}\right)\left(\frac{\beta_{11}^{\mathrm{TE}}}{k(x_{11}'^2 - 1)}\right)} \sum_{\gamma=1}^{2} \sum_{q=0} \sum_{\substack{p=0 \\ p+q\neq 0}} (-1)^{\gamma+1} \epsilon_{pq}$$

$$\cdot\left\{ \left(q\left[\hat{G}_q^{\mathrm{TE}}\right]_{11} \phi_p^{b2} - \left(\frac{pc}{b}\right)\left(\frac{\sin\phi_o}{\phi_o}\right)\frac{x_{11}'^2 \left[\hat{G}_q^{(4)}\right]_{11} \phi_p^{b3}}{\beta_{11}^{\mathrm{TE}} a}\right)\left(\frac{V_{pl}^{\gamma\mathrm{TM}}}{\sqrt{\eta}}\right)\right.$$

$$\left. + \left(\left(\frac{pc}{b}\right)\left[\hat{G}_q^{\mathrm{TE}}\right]_{11} \phi_p^{b2} + q\left(\frac{\sin\phi_o}{\phi_o}\right)\frac{x_{11}'^2 \left[\hat{G}_q^{(4)}\right]_{11} \phi_p^{b3}}{\beta_{11}^{\mathrm{TE}} a}\right)\left(\frac{V_{pl}^{\gamma\mathrm{TE}}}{\sqrt{\eta}}\right)\right\}. \quad \text{(B.7)}$$

Equations (B.5), (B.6), and (B.7) are identical to (B.1), (B.2), and (B.3), respectively. Hence, our objective has been achieved.

References

[1] J. R. Mautz and R. F. Harrington, "Analysis of a TM_{01} circular to TE_{10} rectangular waveguide mode converter," Technical Report SYRU/DECE/TR-89/3, Department of Electrical and Computer Engineering, Syracuse University, Syracuse, NY 13244–1240, Aug. 1989.

[2] J. R. Mautz and R. F. Harrington, "A procedure for numerical evaluation of the performance of a TM_{01} circular to TE_{10} rectangular waveguide mode converter," Technical Report SYRU/DECE/TR-90/3, Department of Electrical and Computer Engineering, Syracuse University, Syracuse, NY 13244–1240, Nov. 1990.

[3] J. R. Mautz and R. F. Harrington, "Computer program for numerical evaluation of the performance of a TM_{01} circular to TE_{10} rectangular waveguide mode converter," Technical Report SYRU/DECE/TR-92/1, Department of Electrical and Computer Engineering, Syracuse University, Syracuse, NY 13244–1240, Jan. 1992.

[4] M. Abramowitz, and I. A. Stegun, *Handbook of Mathematical Functions*. Washington: U. S. Government Printing Office, 1964.

[5] H. B. Dwight, *Tables of Integrals and Other Mathematical Data* (fourth edition). New York: Macmillan, 1961.

Distribution

Administrator
Defense Tech Information Ctr
Attn: DTIC-DDA (2 copies)
Cameron Station, Building 5
Alexandria, VA 22304-6145

Director
Phillips Lab
Attn: PL/WSR, E. Baca
Kirtland AFB
Albuquerque, NM 87117-6008

Clemson University
Dept. of Electrical & Computer Science
Attn: C. H. Butler
Clemson, SC 29634

University of Syracuse
Dept of Electrical & Computer Eng
Attn: R. F. Harrington (5 copies)
Link Hall
Syracuse, NY 13244-1240

University of Utah
Microwave Device & Physical
  Electronics Lab
Attn: J. M. Baird
Joseph Merrill Engineering Building
Salt Lake City, UT 84112

Universityof Delaware
  Ctr for the Mathematics of Waves
Attn: Dept of Mathematical Sciences,
  R. E. Kleinman
Newark, DE 19716

R. C. Hansen, Inc
Attn: R. Hansen
PO Box 570215
Tarzana, CA 91357

U.S. Army Research Lab
Attn: AMSRL-WT, J. Frazier
Aberdeen Proving Ground, MD 21005-5066

U.S. Army Research Lab
Attn: AMSRL-OP-SD-T, Tech Library
  (3 copies)
Attn: AMSRL-OP-SD-TM, Mail & Records
  Mgmt
Attn: AMSRL-OP-SD-TP, Tech Pub
Attn: AMSRL-WT-NA, Chief
Attn: AMSRL-WT-NC, Chief
Attn: AMSRL-WT-NB, Chief
Attn: AMSRL-WT-ND, Chief
Attn: AMSRL-WT-NF, Chief
Attn: AMSRL-WT-NG, Chief
Attn: AMSRL-WT-NH, Chief
Attn: AMSRL-WT-NG, A. Bromborsky
Attn: AMSRL-WT-NH, G. Huttlin
Attn: AMSRL-WT-NH, L. Libelo (10 copies)
Attn: AMSRL-SL-ND, B. Ruth
Attn: AMSRL-WT-N, Director
Attn: AMSRL-WT-NH, M. Litz